Gravitational Production of Non-Minimal Vector Dark Matter

Bohdan Grzadkowski

June 5, 2024

University of Warsaw

based on:

- A. Ahmed, BG, A. Socha, JHEP 02 (2023) 196, e-Print: 2207.11218,
- A. Ahmed, BG, A. Socha, Phys.Lett.B 831 (2022) 137201, e-Print: 2111.06065,
- A. Ahmed, BG, A. Socha, JHEP 08 (2020) 059, e-Print: 2005.01766,
- BG, A. Socha, work in progress.

PLANCK2024 - The 26th International Conference From the Planck Scale to the Electroweak Scale, June, 5th, 2024, Lisbon

Table of Contents

- Background dynamics
- Vector boson dark matter
- Gravitational production of DM
- Summary

Background dynamics

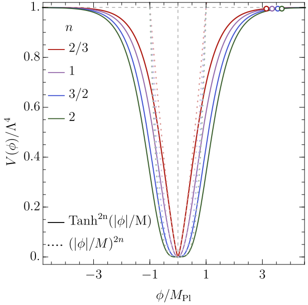
The α-attractor T-model

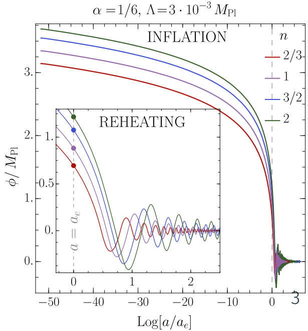
$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{\sqrt{6\alpha}M_{\rm Pl}}\right)$$

$$\simeq \begin{cases} \Lambda^4 & |\phi| \gg M_{\rm Pl} \\ \boxed{\Lambda^4 \left|\dfrac{\phi}{M_{\rm Pl}}\right|^{2n}} & |\phi| \ll M_{\rm Pl} \end{cases},$$

where $n > 0$, $6\alpha = 1$, $\Lambda = 3.0 \times 10^{-3}\, M_{\rm Pl}$





$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

The dynamics of the inflaton field and the scale factor is described by the following classical equations of motion:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$
$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

where $H \equiv \dot{a}/a$ denotes the Hubble parameter.

Assumption: $\rho_X \ll \rho_\phi$

Vector boson dark matter

The FLRW metric:

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2 = a^2(\tau)\left[d\tau^2 - d\vec{x}^2\right]$$

The action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R + \mathcal{L}_\phi + \mathcal{L}_{\mathrm{DM}}\right]$$

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right],$$

$$S_{DM} = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{m_X^2}{2}g^{\mu\nu}X_\mu X_\nu + \right.$$
$$\left. -\frac{\xi_1}{2}g^{\mu\nu}RX_\mu X_\nu + \frac{\xi_2}{2}R^{\mu\nu}X_\mu X_\nu\right\},$$

where $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ with $Z_2 : X_\mu \to -X_\mu$.

O. Özsoy and G. Tasinato, "Vector dark matter, inflation and non-minimal couplings with gravity", 2310.03862,

C. Capanelli, L. Jenks, E.W. Kolb, E. McDonough, "Runaway Gravitational Production of Dark Photons", 2403.15536,

BG., A. Socha, "Purely gravitational production of dark vectors non-minimally coupled to gravity", in progress,

A. Ahmed, BG, A. Socha, "Gravitational production of vector dark matter", JHEP 08 (2020) 059, 2005.01766.

Gravitational production of DM

$$X_\mu(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} X_\mu(\tau, \vec{k}) e^{i\vec{k}\cdot\vec{x}}, \qquad \vec{X}(t, \vec{k}) = \sum_{\lambda=\pm,L} \vec{\epsilon}_\lambda(\vec{k}) X_\lambda(t, \vec{k}),$$

$$S_T = \sum_{T=\pm} \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2}|X_T'(\tau, \vec{k})|^2 - \frac{1}{2}[k^2 + a^2 m^2_{\text{eff,x}}(a)]|X_T(\tau, \vec{k})|^2 \right\},$$

$$S_L = \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} \frac{1}{A_L^2(a, k)} |X_L'(\tau, \vec{k})|^2 - \frac{1}{2} a^2 m^2_{\text{eff,x}}(a)|X_L(\tau, \vec{k})|^2 \right\},$$

where $k^2 \equiv |\vec{k}|^2$, and

$$A_L^2(a, k) \equiv \frac{k^2 + a^2 m^2_{\text{eff,t}}(a)}{a^2 m^2_{\text{eff,t}}(a)},$$

$$m^2_{\text{eff,t}}(a) \equiv m_X^2 - \xi_1 R(a) + \frac{1}{2}\xi_2 R(a) + 3\xi_2 H^2(a),$$

$$m^2_{\text{eff,x}}(a) \equiv m_X^2 - \xi_1 R(a) + \frac{1}{6}\xi_2 R(a) - \xi_2 H^2(a).$$

$$m^2_{\text{eff,t}}(a) = m_X^2 - 3\left[\left(\xi_1 - \frac{1}{2}\xi_2\right)(3w(a) - 1) - \xi_2\right] H^2(a),$$
$$m^2_{\text{eff,x}}(a) = m_X^2 - \left[3\left(\xi_1 - \frac{1}{6}\xi_2\right)(3w(a) - 1) + \xi_2\right] H^2(a).$$

where

$$w(a) \equiv \frac{p(a)}{\rho(a)} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}, \qquad w(a) \in [-1, 1]$$

- For minimal couplings, i.e. $\xi_1 = \xi_2 = 0$: $m^2_{\text{eff,x}}(a) = m^2_{\text{eff,t}}(a) = m_X^2$.
- During inflation (dS)

$$m^2_{\text{eff,x}}(a) = m^2_{\text{eff,t}}(a) = m_X^2 + 3(4\xi_1 - \xi_2)H^2(a) \simeq \text{const.}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

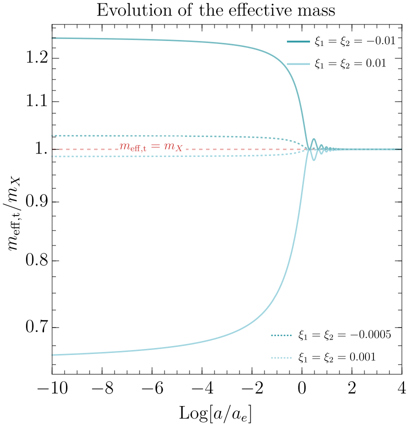


Figure 1:

$$s(a) \equiv \text{sign}\left\{\frac{k^2 + a^2 m^2_{\text{eff,t}}(a)}{a^2 m^2_{\text{eff,t}}(a)}\right\}$$

To avoid ghost instability $s(a) > 0$ for any a: $\rightsquigarrow m^2_{\text{eff,t}}(a) > 0$

$$\Downarrow$$

$$f(w(a), \xi_1, \xi_2) \leq \left(\frac{m_X}{H_e}\right)^2 \equiv \eta_e^{-1}$$

with

$$f(w(a), \xi_1, \xi_2) \equiv 3\left[\left(\xi_1 - \frac{1}{2}\xi_2\right)(3w(a) - 1) - \xi_2\right] \text{ for } w(a) \in [-1, 1]$$

For

$$\xi_1 = \frac{1}{2}\xi_2.$$

The Lagrangian density reads

$$\sqrt{-g}\mathcal{L}_{\mathrm{X}}^{\mathrm{NM}} = \sqrt{-g}\left[-\frac{\xi_1}{2}Rg_{\mu\nu}X^\mu X^\nu + \frac{\xi_2}{2}R_{\mu\nu}X^\mu X^\nu\right] = \sqrt{-g}\frac{1}{2}\xi_2 G_{\mu\nu}X^\mu X^\nu,$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$.

$$m_{\mathrm{eff,x}}^2(a)\Big|_{\xi_1=\xi_2/2} = m_X^2 - w(a)3\xi_2 H^2(a)$$

$$m_{\mathrm{eff,t}}^2(a)\Big|_{\xi_1=\xi_2/2} = m_X^2 + 3\xi_2 H^2(a)$$

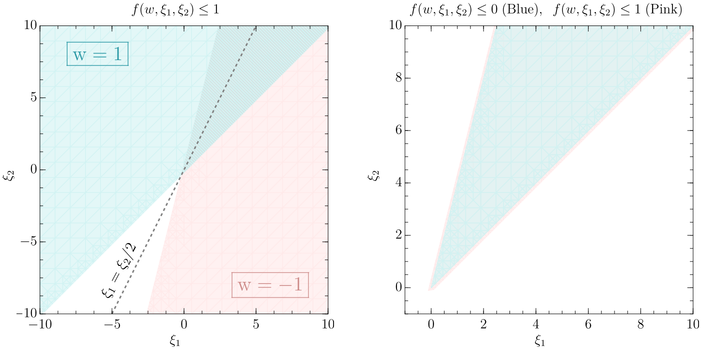


Figure 2: Left: Region in the $\xi_1 - \xi_2$ parameter space satisfying $f(w(a), \xi_1, \xi_2) \lesssim 1$, i.e. for $\eta_e = 1$, with two limiting choices of the equation-of-state parameter $w = -1$ (light pink region) and $w = 1$ (light cyan region). Right: Values of $\xi_1 - \xi_2$ ensuring the positivity of $m^2_{\mathrm{eff,t}}(a)$ for two values of $\eta_e^{-1} \in \{0, 1\}$.

$$X_L(\tau, \vec{k}) = A_{\rm L}(a,k)\mathcal{X}_L$$

Integrating by parts and dropping the boundary term

$$S_{\rm L} = \frac{1}{2}\int d\tau \int \frac{d^3k}{(2\pi)^3}\times$$
$$\times \left\{ |\mathcal{X}_{\rm L}'(\tau,\vec{k})|^2 - \left[a^2 m^2_{\rm eff,X}(a) A_{\rm L}^2(a,k) + \frac{A_{\rm L}''(a,k)}{A(a,k)} - 2\left(\frac{A_{\rm L}'(a,k)}{A_{\rm L}(a,k)}\right)^2\right] |\mathcal{X}_{\rm L}(\tau,\vec{k})|^2 \right\}$$

$$X_{\rm T}''(\tau,\vec{k}) + \omega_{\rm T}^2(\tau,k) X_{\rm T}(\tau,\vec{k}) = 0,$$
$$\mathcal{X}_{\rm L}''(\tau,\vec{k}) + \omega_{\rm L}^2(\tau,k) \mathcal{X}_{\rm L}(\tau,\vec{k}) = 0,$$

where the time-dependent frequencies are defined as

$$\omega_{\rm T}^2(\tau,k) \equiv k^2 + a^2 m^2_{\rm eff,X}(a),$$
$$\omega_{\rm L}^2(\tau,k) \equiv a^2 m^2_{\rm eff,X}(a) A_{\rm L}^2(a,k) + \frac{A_{\rm L}''(a,k)}{A_{\rm L}(a,k)} - 2\left(\frac{A_{\rm L}'(a,k)}{A_{\rm L}(a,k)}\right)^2$$

$$\omega_{\mathrm{L}}^2(\tau, k) = k^2 \frac{m_{\mathrm{eff,x}}^2}{m_{\mathrm{eff,t}}^2} + a^2 m_{\mathrm{eff,x}}^2(a) + \\ - \frac{k^2}{k^2 + a^2 m_{\mathrm{eff,t}}^2(a)} \left[\frac{a''}{a} + \frac{m''_{\mathrm{eff,t}}}{m_{\mathrm{eff,t}}} + 2\frac{a'}{a}\frac{m'_{\mathrm{eff,t}}}{m_{\mathrm{eff,t}}} - 3\frac{(a' m_{\mathrm{eff,t}} + m'_{\mathrm{eff,t}} a)^2}{k^2 + a^2 m_{\mathrm{eff,t}}^2(a)} \right]$$

with

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

and

$$H = \frac{\dot{a}}{a}$$

$$\omega_{\mathrm{T}}^2(a,k) = k^2 + a^2 m_X^2 - a^2 H^2(a)\left[3(3w(a)-1)\left(\xi_1 - \frac{1}{6}\xi_2\right) + \xi_2\right]$$

For the minimal coupling, i.e. $\xi_1 = \xi_2 = 0$, the frequency recovers the standard formula

$$\omega_{\mathrm{L}}^2(\tau,k) = \omega_{\mathrm{L}}^2(\tau,k)\,\big|_{\xi_1=\xi_2=0} = k^2 + a^2 m_X^2 - \frac{k^2}{k^2 + a^2 m_X^2}\left[\frac{a''}{a} - 3\frac{a'^2 m_X^2}{k^2 + a^2 m_{\mathrm{X}}^2}\right]$$

see also

- A. Ahmed, B.G. and A. Socha, “Gravitational production of vector dark matter,” JHEP **08** (2020), 059
- E. W. Kolb and A. J. Long, “Completely dark photons from gravitational particle production during the inflationary era,” JHEP **03** (2021), 283

UV behaviour, i.e. $k^2 \to \infty$:

$$\omega_T^2(a,k) \to k^2, \qquad \omega_L^2(a,k) \to k^2 \frac{m^2_{\text{eff,x}}(a)}{m^2_{\text{eff,t}}(a)}$$

$$\frac{m^2_{\text{eff,x}}(a)}{m^2_{\text{eff,t}}(a)} \lesseqgtr 0$$

$$\frac{m^2_{\text{eff,x}}(a)}{m^2_{\text{eff,t}}(a)} < 0 \Rightarrow \text{massive creation of short-wavelength modes}$$

Remark:

- during dS inflation $m^2_{\text{eff,x}}(a) = m^2_{\text{eff,t}}(a)$, therefore for $k^2 \to \infty$ $\omega_T^2(a,k) = \omega_L^2(a,k) = k^2$, i.e. no massive production of short-wavelength modes, Credibility might be restored:

- One could impose the positivity condition on $m^2_{\text{eff,x}}(a)$ analogously to $m^2_{\text{eff,t}}(a)$:

$$\tilde{f}(w, \xi_1, \xi_2) \lesssim \left(\frac{m_X}{H(a_e)}\right)^2 = \eta_e^{-1},$$

with

$$\tilde{f}(w, \xi_1, \xi_2) \equiv 3\left[3w(a) - 1\right]\left(\xi_1 - \frac{1}{6}\xi_2\right) + \xi_2.$$

- For $m_X \to 0$ and $\xi_1, \xi_2 \neq 0$ there is no region such that $m^2_{\text{eff,x}}(a) > 0$ and $m^2_{\text{eff,t}}(a) > 0$ for arbitrary $w \in [-1, 1]$.
- If $m^2_{\text{eff,x}}(a) > 0$ for any a, then $\omega^2_{\text{T}}(\tau, k) \equiv k^2 + a^2 m^2_{\text{eff,x}}(a) > 0$, so no tachyonic production of X_T.

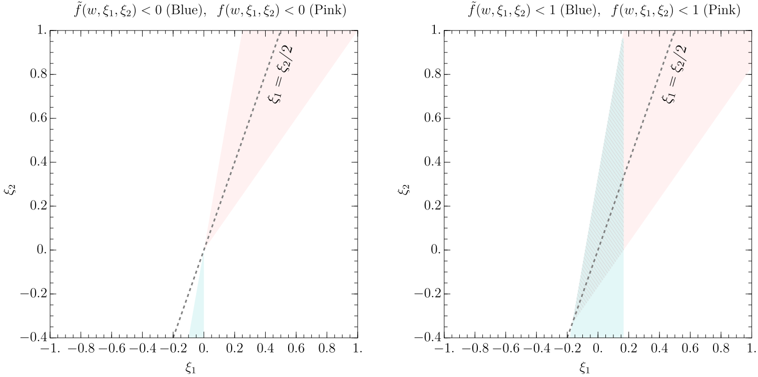


Figure 3: Left: Region in the $\xi_1 - \xi_2$ parameter space satisfying $f(w(a), \xi_1, \xi_2) \lesssim \eta_e^{-1}$ and $\tilde{f}(w(a), \xi_1, \xi_2) \lesssim \eta_e^{-1}$, for $\eta_e^{-1} = 0$ (left panel) and $\eta_e^{-1} = 1$ (right panel).

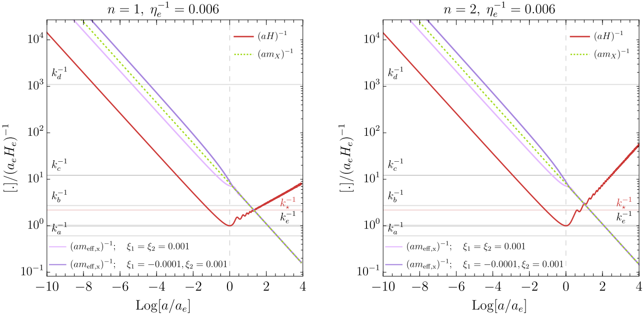


Figure 4: Evolution of various length scales, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

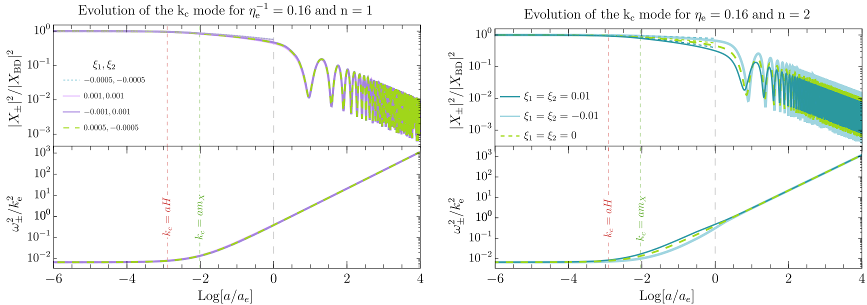


Figure 5: Evolution of the k_c momentum mode of the transversely polarized vectors with mass $m_X = 5 \cdot 10^{12}\,\mathrm{GeV}$ for different non-minimal couplings for $n = 1$ (left) and $n = 2$ (right). Lower panels: Evolution of the transverse frequency $\omega_\pm^2/k_e^2$, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

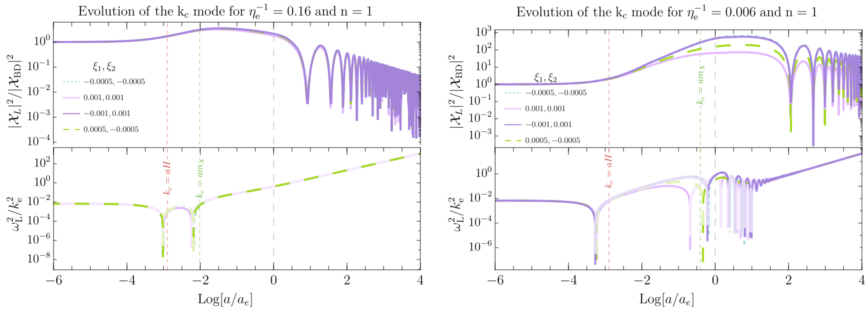


Figure 6: Evolution of the k_c momentum mods of the redefined longitudinal polarization with mass $m_X = 5 \cdot 10^{12}\,\mathrm{GeV}$ (left) and $m_X = 10^{12}\,\mathrm{GeV}$ (right) for different non-minimal couplings assuming quadratic inflaton potential during reheating, i.e., $n = 1$. Lower panels: Evolution of longitudinal frequency $\omega_{\mathrm{L}}^2/k_e^2$, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

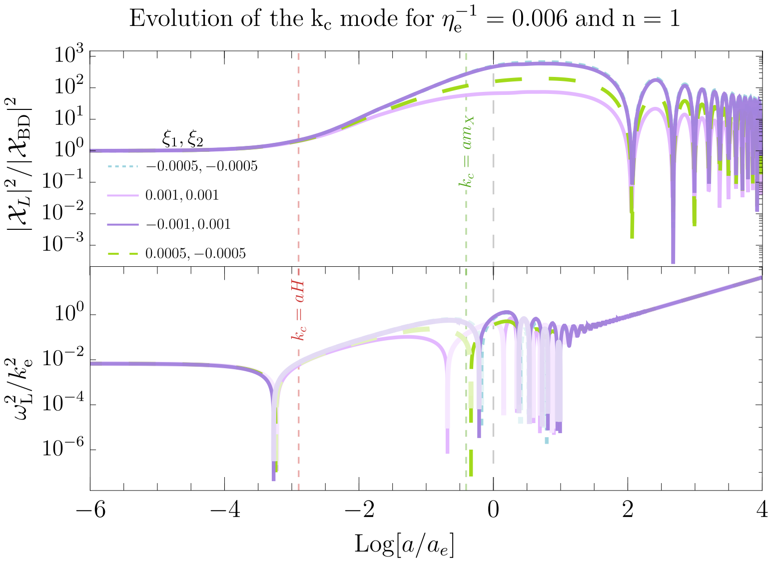


Figure 7: Evolution of the k_c momentum mod of the redefined longitudinal polarization with mass $m_X = 5 \cdot 10^{12}\,\mathrm{GeV}$ (left) and $m_X = 10^{12}\,\mathrm{GeV}$ (right) for different non-minimal couplings assuming quadratic inflaton potential during reheating, i.e., $n = 1$. Lower panel: Evolution of longitudinal frequency ω_L^2/k_e^2, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

$$T_{\mu\nu} := \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{M}}}{\delta g^{\mu\nu}}, \qquad T^{\mathrm{X}}_{\mu\nu} = T^{\mathrm{M}}_{\mu\nu} + T^{\xi_1}_{\mu\nu} + T^{\xi_2}_{\mu\nu}$$

$$T^{\mathrm{M}}_{\mu\nu} = g_{\mu\nu} \left(\frac{1}{4} g^{\rho\sigma} g^{\alpha\beta} X_{\rho\alpha} X_{\sigma\beta} - \frac{m_X^2}{2} g^{\alpha\beta} X_\alpha X_\beta \right) - g^{\alpha\beta} X_{\mu\alpha} X_{\nu\beta} + m_X^2 X_\mu X_\nu$$

$$T^{\xi_1}_{\mu\nu} = \xi_1 \Big[- R X_\mu X_\nu - G_{\mu\nu} g^{\rho\sigma} X_\rho X_\sigma +$$

$$- g_{\mu\nu} g^{\rho\sigma} g^{\alpha\beta} \nabla_\sigma \nabla_\rho (X_\alpha X_\beta) + g^{\rho\sigma} \nabla_\mu \nabla_\nu (X_\rho X_\sigma) \Big]$$

$$T^{\xi_2}_{\mu\nu} = \frac{\xi_2}{2} \Big[- g_{\mu\nu} g^{\alpha\rho} g^{\beta\sigma} R_{\rho\sigma} X_\alpha X_\beta + 2 g^{\rho\sigma} R_{\nu\sigma} X_\mu X_\rho + 2 g^{\rho\sigma} R_{\mu\sigma} X_\nu X_\rho +$$

$$+ g^{\rho\sigma} \nabla_\rho \nabla_\sigma (X_\mu X_\nu) + g_{\mu\nu} g^{\lambda\rho} g^{\kappa\sigma} \nabla_\lambda \nabla_\kappa (X_\rho X_\sigma) - g^{\lambda\sigma} \nabla_\mu \nabla_\sigma (X_\lambda X_\nu) +$$

$$- g^{\lambda\sigma} \nabla_\nu \nabla_\sigma (X_\lambda X_\mu) \Big]$$

$$\hat{\vec{X}}(\tau,\vec{x}) = \sum_{\lambda=\pm,L}\int \frac{d^3k}{(2\pi)^3}\vec{\epsilon}_\lambda(\vec{k})e^{i\vec{k}\cdot\vec{x}}\hat{\mathcal{X}}_\lambda(\tau,k),$$

$$\hat{\mathcal{X}}_\lambda(\tau,k) \equiv \hat{a}_\lambda(\vec{k})\mathcal{X}_\lambda(\tau,k) + \hat{a}^\dagger_\lambda(-\vec{k})\mathcal{X}^\star_\lambda(\tau,k)$$

The power spectra:

$$\langle\hat{\mathcal{X}}_\lambda(\tau,k)\cdot\hat{\mathcal{X}}_{\lambda'}(\tau,q)\rangle = \delta_{\lambda\lambda'}(2\pi)^3\delta^{(3)}(\vec{k}+\vec{q})\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{X}_\lambda}(\tau,k)$$

$$\langle\hat{\mathcal{X}}'_\lambda(\tau,k)\cdot\hat{\mathcal{X}}'_{\lambda'}(\tau,q)\rangle = \delta_{\lambda\lambda'}(2\pi)^3\delta^{(3)}(\vec{k}+\vec{q})\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{X}'_\lambda}(\tau,k)$$

$$\langle\hat{\mathcal{X}}_\lambda(\tau,k)\cdot\hat{\mathcal{X}}'_{\lambda'}(\tau,q)\rangle + \langle\hat{\mathcal{X}}'_\lambda(\tau,k)\cdot\hat{\mathcal{X}}_{\lambda'}(\tau,q)\rangle = \delta_{\lambda\lambda'}(2\pi)^3\delta^{(3)}(\vec{k}+\vec{q})\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{X}_\lambda\mathcal{X}'_\lambda}(\tau,k)$$

where $\lambda,\lambda' = \pm, \mathrm{L}$

$$\langle\hat{\rho}_X\rangle = \langle\hat{\rho}_\mathrm{L}\rangle + \langle\hat{\rho}_\pm\rangle,$$

where

$$\langle\hat{\rho}_\mathrm{L}\rangle = \langle\hat{\rho}^\mathrm{M}_\mathrm{L}\rangle + \langle\hat{\rho}^{\xi_1}_\mathrm{L}\rangle + \langle\hat{\rho}^{\xi_2}_\mathrm{L}\rangle,$$

$$\langle\hat{\rho}_\pm\rangle = \langle\hat{\rho}^\mathrm{M}_\pm\rangle + \langle\hat{\rho}^{\xi_1}_\pm\rangle + \langle\hat{\rho}^{\xi_2}_\pm\rangle$$

$$\langle \hat{\rho}_{\pm}^{M} \rangle = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k^3} \left\{ \mathcal{P}_{\mathcal{X}'_{\pm}}(\tau, k) + (k^2 + a^2 m_X^2) \mathcal{P}_{\mathcal{X}_{\pm}}(\tau, k) \right\}$$

$$\langle \hat{\rho}_{\pm}^{\xi_1} \rangle = \frac{\xi_1}{a^4} \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k^3} \left\{ -3a^2 H^2 \mathcal{P}_{\mathcal{X}_{\pm}}(\tau, k) + 3aH \mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}} \right\}$$

$$\langle \hat{\rho}_{\pm}^{\xi_2} \rangle = \frac{\xi_2}{a^4} \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k^3} \left\{ 2a^2 H^2 \mathcal{P}_{\mathcal{X}_{\pm}}(\tau, k) - 3aH \mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}} \right\}$$

Spectral energy densities

$$\frac{\frac{d\langle \hat{\rho}_{\pm}^{M} \rangle}{d\ln k}}{\left.\frac{d\langle \hat{\rho}_{\pm} \rangle}{d\ln k}\right|_{k=k_e, a=a_e}} \propto \frac{1}{2a^4} \left\{ \mathcal{P}_{\mathcal{X}'_{\pm}}(\tau, k) + (k^2 + a^2 m_X^2) \mathcal{P}_{\mathcal{X}_{\pm}}(\tau, k) \right\}$$

$$\frac{\frac{d\langle \hat{\rho}_{\pm}^{\xi_1} \rangle}{d\ln k}}{\left.\frac{d\langle \hat{\rho}_{\pm} \rangle}{d\ln k}\right|_{k=k_e, a=a_e}} \propto \frac{\xi_1}{a^4} \left\{ -3a^2 H^2 \mathcal{P}_{\mathcal{X}_{\pm}}(\tau, k) + 3aH \mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}} \right\}$$

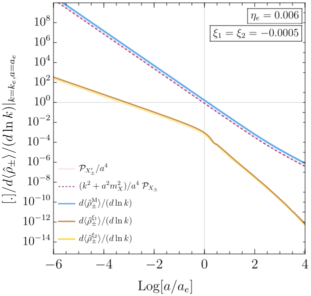
$$\frac{\frac{d\langle \hat{\rho}_{\pm}^{\xi_2} \rangle}{d\ln k}}{\left.\frac{d\langle \hat{\rho}_{\pm} \rangle}{d\ln k}\right|_{k=k_e, a=a_e}} \propto \frac{\xi_2}{a^4} \left\{ 2a^2 H^2 \mathcal{P}_{\mathcal{X}_{\pm}}(\tau, k) - 3aH \mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}} \right\}$$

$$\frac{\frac{d\langle\hat{\rho}^{\mathrm{M}}_{\pm}\rangle}{d\ln k}}{\frac{d\langle\hat{\rho}_{\pm}\rangle}{d\ln k}\Big|_{k=k_e,a=a_e}} \propto \frac{1}{2a^4}\left\{\mathcal{P}_{\mathcal{X}'_{\pm}}(\tau,k) + (k^2 + a^2 m_X^2)\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k)\right\}$$

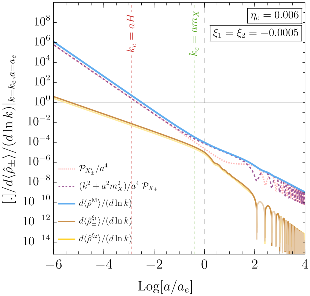
$$\frac{\frac{d\langle\hat{\rho}^{\xi_1}_{\pm}\rangle}{d\ln k}}{\frac{d\langle\hat{\rho}_{\pm}\rangle}{d\ln k}\Big|_{k=k_e,a=a_e}} \propto \frac{\xi_1}{a^4}\left\{-3a^2H^2\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k) + 3aH\mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}}\right\}$$

$$\frac{\frac{d\langle\hat{\rho}^{\xi_2}_{\pm}\rangle}{d\ln k}}{\frac{d\langle\hat{\rho}_{\pm}\rangle}{d\ln k}\Big|_{k=k_e,a=a_e}} \propto \frac{\xi_2}{a^4}\left\{2a^2H^2\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k) - 3aH\mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}}\right\}$$

Evolution of the spectral energy density for the k_a mode



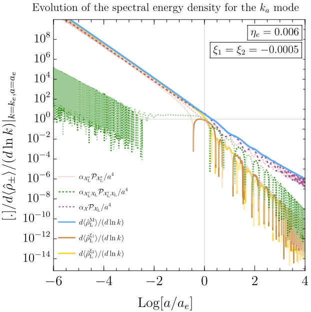
Evolution of the spectral energy density for the k_c mode

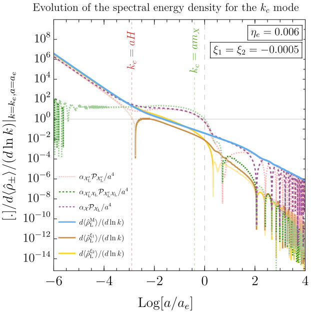


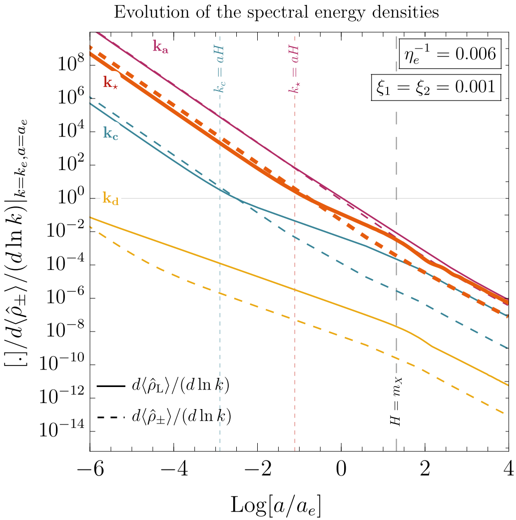
$$\langle \hat{\rho}_{\mathrm{L}}^{\mathrm{M}} \rangle = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k^3} \left\{ \frac{a^2 m_X^2}{k^2 + a^2 m_X^2} A_{\mathrm{L}} \left[A_{\mathrm{L}} \mathcal{P}_{\mathcal{X}'_{\mathrm{L}}} + A'_{\mathrm{L}} \mathcal{P}_{\mathcal{X}_{\mathrm{L}} \mathcal{X}'_{\mathrm{L}}} \right] + \cdots \right\}$$

$$\langle \hat{\rho}_{\mathrm{L}}^{\xi_1} \rangle = \frac{\xi_1}{a^4} \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k^3} \left\{ 3(aH)^2 (-3w + 4) \frac{k^2}{(k^2 + a^2 m_X^2)^2} A_{\mathrm{L}}^2 \mathcal{P}_{\mathcal{X}'_{\mathrm{L}}} + \cdots \right\}$$

$$\langle \hat{\rho}_{\mathrm{L}}^{\xi_2} \rangle = \frac{\xi_2}{2a^4} \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k^3} \left\{ 3(aH)^2 (3w - 1) \frac{k^2}{(k^2 + a^2 m_X^2)^2} A_{\mathrm{L}}^2 \mathcal{P}_{\mathcal{X}'_{\mathrm{L}}} + \cdots \right\}$$







Evolution of the spectral energy densities
$\mathbf{k_a}$
$\mathbf{k_\star}$
$\mathbf{k_c}$
$\mathbf{k_d}$
$k_c = aH$
$k_\star = aH$
$H = m_X$
$\eta_e^{-1} = 0.006$
$\xi_1 = \xi_2 = 0.001$
$d\langle\hat{\rho}_{\rm L}\rangle/(d\ln k)$
$d\langle\hat{\rho}_\pm\rangle/(d\ln k)$
$[\cdot]/d\langle\hat{\rho}_\pm\rangle/(d\ln k)|_{k=k_e,a=a_e}$
$\mathrm{Log}[a/a_e]$

Figure 8:

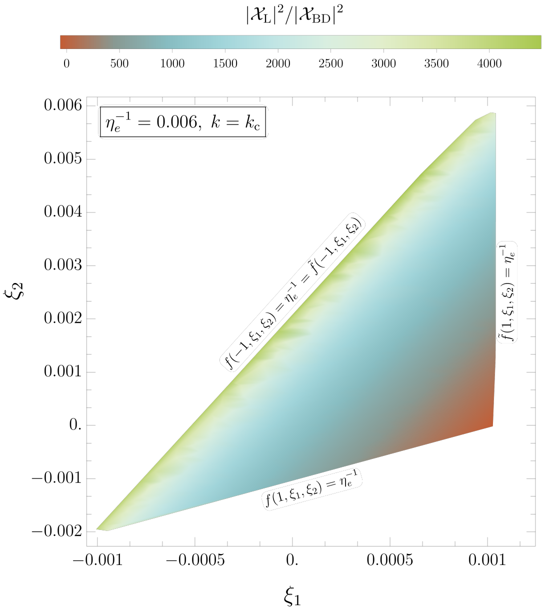


Figure 9: The amplitude squared of the longitudinal polarization normalized to the Bunch-Davies value for different choices of the non-minimal coupling ξ_1, ξ_1 satisfying constraints $f(w(a), \xi_1, \xi_2) \gtrsim \eta_e^{-1}$, and $\tilde{f}(w(a), \xi_1, \xi_2) \gtrsim \eta_e^{-1}$.

Summary

- Gravitational production of Abelian massive gauge fields, candidates for dark matter, that are coupled non-minimally to gravity has been discussed.
- The α-attractor T-model potential for the inflaton field has been adopted:

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{\sqrt{6\alpha}M_{\rm Pl}}\right) \simeq \begin{cases} \Lambda^4 & |\phi| \gg M_{\rm Pl} \\ \boxed{\Lambda^4 \left|\frac{\phi}{M_{\rm Pl}}\right|^{2n}} & |\phi| \ll M_{\rm Pl} \end{cases},$$

- Energy density corresponding to various polarization components of the vector field have been calculated.
- Spectator vector X_μ: $\rho_X \ll \rho_\phi$

- It has been shown that the presence of the non-minimal couplings may imply a massive, tachyonic production of high-momentum modes of the gauge field.
- For $m_X \to 0$ and $\xi_1, \xi_2 \neq 0$ there is no region such that $m^2_{\text{eff,x}}(a) > 0$ and $m^2_{\text{eff,t}}(a) > 0$ for arbitrary $w \in [-1, 1]$.
- Oscylatory tachyonic production during reheating have been observed.

Backup slides

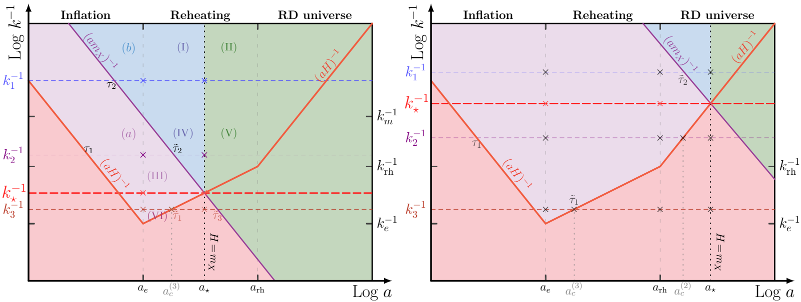


Figure 10: Evolution of various cosmological distances during and after inflation for heavy vector DM i.e. $H_{\rm rh} \leq m_X < H_{\rm I}$ (left diagram) and light vector DM $m_X < H_{\rm rh}$ (right diagram). The red region corresponds to modes with wavevector in range $am_X, aH \ll k$, purple refers to the region where $am_X < k < aH$, blue corresponds to the condition $k < am_X < aH$ and in the green region $aH, k \ll am_X$. Here $a_c = k/H$ refers to the second *horizon crossing*, $a_\star \equiv a(\tau_\star)$, $k_m \equiv a_e m_X$, $k_\star = a_\star m_X$, $k_e \equiv a_e H_{\rm I}$ and $k_{\rm rh} \equiv a_{\rm rh} H_{\rm rh}$. The plot assumes $-1/3 < w < 1/3$ during the reheating phase.

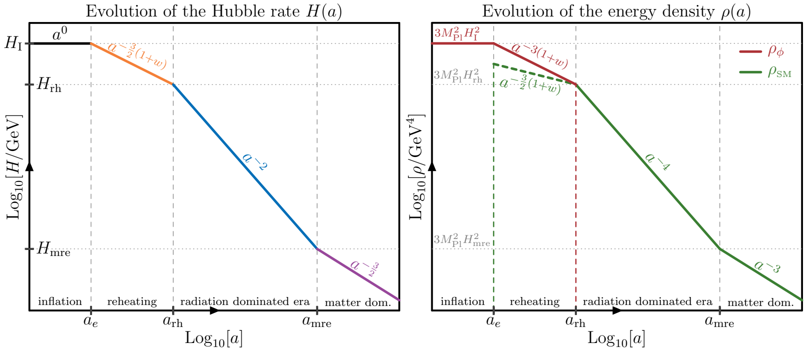


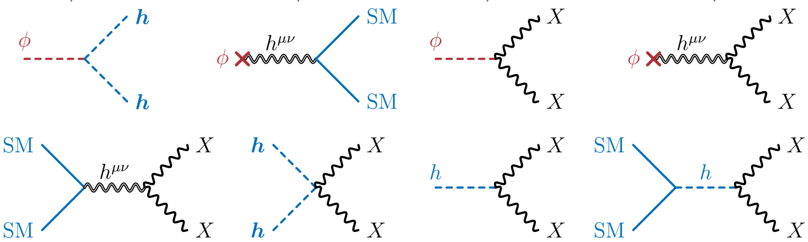
Figure 11: The cosmological evolution of the Hubble rate $H(a)$ (left-panel) and energy density $\rho(a)$ (right-panel) as a function of the scale factor a. The scale factor at the end of inflation a_e, end of reheating $a_{\rm rh}$, and matter-radiation equality $a_{\rm mre}$ are represented as gray dashed vertical lines.

$$\dot{\rho}_\phi + \frac{6n}{n+1} H \rho_\phi = -\boxed{\langle \Gamma_\phi \rangle}\, \rho_\phi$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \langle \Gamma_{\phi\to {\rm SM\,SM}} \rangle \rho_\phi - 2\langle E_X \rangle \mathcal{S}_{\rm SM} - \langle E_{h_0} \rangle \mathcal{D}_{h_0}$$

$$\dot{n}_X + 3Hn_X = \mathcal{D}_\phi + \mathcal{S}_\phi + \mathcal{S}_{\rm SM} + \mathcal{D}_{h_0}$$

with the Hubble rate $H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\rho_\phi + \rho_{\rm SM} + \rho_X\right)$

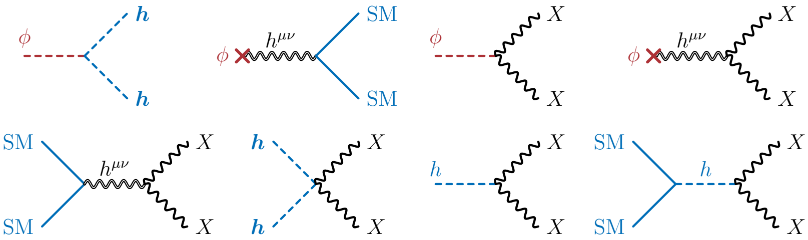


$$\dot{\rho}_\phi + \frac{6n}{n+1} H \rho_\phi = -\langle \Gamma_\phi \rangle \rho_\phi$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \boxed{\langle \Gamma_{\phi \to {\rm SM\,SM}} \rangle} \rho_\phi - 2\langle E_X \rangle \mathcal{S}_{\rm SM} - \langle E_{h_0} \rangle \mathcal{D}_{h_0}$$

$$\dot{n}_X + 3Hn_X = \mathcal{D}_\phi + \mathcal{S}_\phi + \mathcal{S}_{\rm SM} + \mathcal{D}_{h_0}$$

with the Hubble rate $H^2 = \frac{1}{3M_{\rm Pl}^2}(\rho_\phi + \rho_{\rm SM} + \rho_X)$

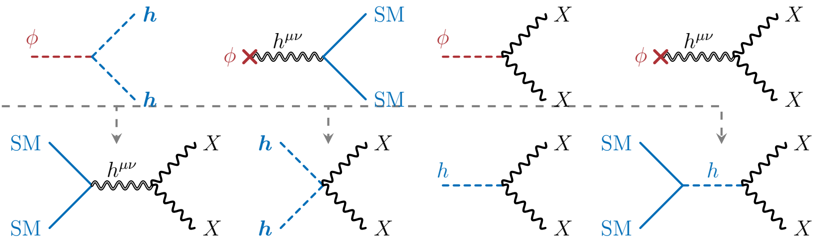


$$\dot{\rho}_\phi + \frac{6n}{n+1} H\rho_\phi = -\langle \Gamma_\phi \rangle \rho_\phi$$

$$\dot{\rho}_{\mathrm{SM}} + 4H\rho_{\mathrm{SM}} = \langle \Gamma_{\phi \to \mathrm{SM\,SM}} \rangle \rho_\phi - 2\langle E_X \rangle \boxed{\mathcal{S}_{\mathrm{SM}}} - \langle E_{h_0} \rangle \mathcal{D}_{h_0}$$

$$\dot{n}_X + 3Hn_X = \mathcal{D}_\phi + \mathcal{S}_\phi + \boxed{\mathcal{S}_{\mathrm{SM}}} + \mathcal{D}_{h_0}$$

with the Hubble rate $H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}(\rho_\phi + \rho_{\mathrm{SM}} + \rho_X)$

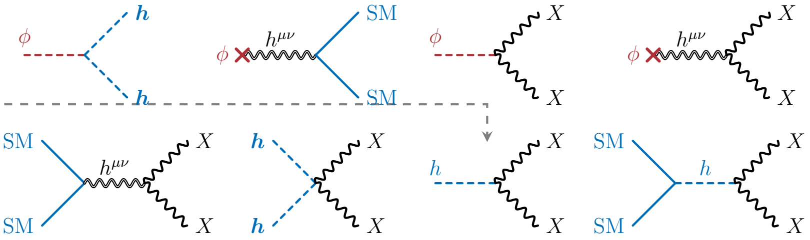


$$\dot{\rho}_\phi + \frac{6n}{n+1} H\rho_\phi = -\langle \Gamma_\phi \rangle \rho_\phi$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \langle \Gamma_{\phi \to {\rm SM\,SM}} \rangle \rho_\phi - 2\langle E_X \rangle \mathcal{S}_{\rm SM} - \langle E_{h_0} \rangle \boxed{\mathcal{D}_{h_0}}$$

$$\dot{n}_X + 3Hn_X = \mathcal{D}_\phi + \mathcal{S}_\phi + \mathcal{S}_{\rm SM} + \boxed{\mathcal{D}_{h_0}}$$

with the Hubble rate $H^2 = \frac{1}{3M_{\rm Pl}^2}(\rho_\phi + \rho_{\rm SM} + \rho_X)$

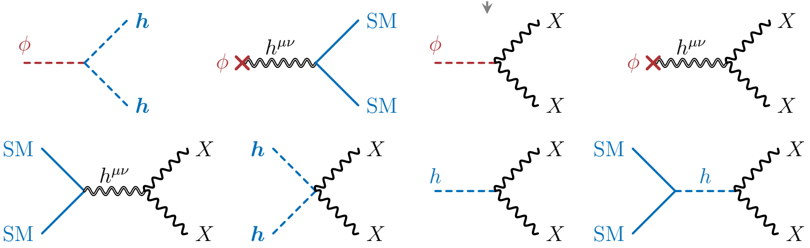


$$\dot{\rho}_\phi + \frac{6n}{n+1} H\rho_\phi = -\langle \Gamma_\phi \rangle \rho_\phi$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \langle \Gamma_{\phi\to {\rm SM\,SM}} \rangle \rho_\phi - 2\langle E_X \rangle \mathcal{S}_{\rm SM} - \langle E_{h_0} \rangle \mathcal{D}_{h_0}$$

$$\dot{n}_X + 3Hn_X = \boxed{\mathcal{D}_\phi} + \mathcal{S}_\phi + \mathcal{S}_{\rm SM} + \mathcal{D}_{h_0}$$

with the Hubble rate $H^2 = \frac{1}{3M_{\rm Pl}^2}(\rho_\phi + \rho_{\rm SM} + \rho_X)$



Quantum particle production from a vacuum in a classical inflaton/gravitational background

For the interactions proportional to the $\phi = \varphi \cdot \mathcal{P}$ term, the lowest-order non-vanishing S-matrix element takes the form

$$\mathcal{S}_{if}^{(1)} = \sum_k \mathcal{P}_k \langle f| \int d^4 x \varphi(t)\, e^{-ik\omega t} \mathcal{L}_{\rm int}(x)\,|i\rangle$$

where

$$|i\rangle \equiv |0\rangle \,, \qquad |f\rangle \equiv \hat{a}_f^\dagger \hat{a}_f^\dagger\, |0\rangle\,.$$

If the envelope $\varphi(t)$ varies on the time-scale much longer than the time-scale relevant for processes of particle creation, the S-matrix element can be written as

$$S_{if}^{(1)} = i\varphi(t) \sum_k \mathcal{P}_k \mathcal{M}_{0\to f}(k) \times (2\pi)^4 \delta(k\omega - 2E_f)\delta^3(p_{f_1} + p_{f_2}).$$

Higher dimensional operators in the dark matter production

$$\mathcal{L}_{\rm int} \supset -\frac{1}{2}\frac{\mathcal{C}_X^{\phi}}{M_{\rm Pl}}\phi\,|D_\mu\Phi|^2 - \frac{1}{2}\frac{\mathcal{C}_X^{\rm h}}{M_{\rm Pl}^2}|\mathrm{h}|^2|D_\mu\Phi|^2,$$

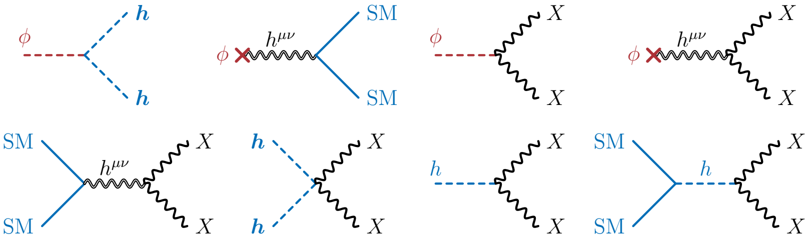
where $D_\mu\Phi = (\partial_\mu - ig_X X_\mu)\Phi$ is the covariant derivative of the Φ field and g_X is the $U(1)_X$ gauge coupling.

$$\mathrm{h} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} h_2 + ih_3 \\ h_0 + ih_1 \end{pmatrix},$$

$$\Downarrow$$

$$\mathcal{L}_{\rm int} = \frac{\mathcal{C}_X^{\phi}\, m_X^2}{2M_{\rm Pl}}\phi X_\mu X^\mu + \frac{\mathcal{C}_X^{\rm h}\, m_X^2}{2M_{\rm Pl}^2}X_\mu X^\mu|\mathrm{h}|^2,$$

$$\mathcal{L}_{\text{int}} = -\left\{ \boxed{g_{h\phi} M_{\text{Pl}}\, \phi |\mathsf{h}|^2} + \frac{h^{\mu\nu}}{M_{\text{Pl}}} \left[T^{\phi}_{\mu\nu} + T^{\text{DM}}_{\mu\nu} + T^{\text{SM}}_{\mu\nu} \right] + \frac{\mathcal{C}^{\phi}_{X}\, m_X^2}{2 M_{\text{Pl}}} \phi X_\mu X^\mu + \frac{\mathcal{C}^{\mathsf{h}}_{X}\, m_X^2}{2 M_{\text{Pl}}^2} |\mathsf{h}|^2 X_\mu X^\mu \right\},$$



Generalized Stuckelberg action

$$\mathcal{S}_X = \int d^4x\sqrt{-g}$$
$$\Bigg\{-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{1}{2}g^{\mu\nu}(\partial_\nu\Phi_X + m_X X_\nu)(\partial_\mu\Phi_X + m_X X_\mu)$$
$$+ \frac{1}{2}\left(1 - \frac{R}{m_1^2}\right)g^{\mu\nu}(\partial_\mu\Phi_1 + |\xi_1|^{1/2}m_1 X_\mu)(\partial_\nu\Phi_1 + |\xi_1|^{1/2}m_1 X_\nu)$$
$$+ \frac{1}{2}\left(g^{\mu\nu} + \frac{R^{\mu\nu}}{m_2^2}\right)(\partial_\mu\Phi_2 + |\xi_2|^{1/2}m_2 X_\mu)(\partial_\nu\Phi_2 + |\xi_2|^{1/2}m_2 X_\nu)\Bigg\},$$

where Φ_i ($i = X, 1, 2$) stand for the Stuckelberg real scalar fields. The above action is invariant under the following three local $U_X(1)$ transformations

$$X_\mu(x) \to X'_\mu(x) = X_\mu(x) + \partial_\mu\lambda_i(x),$$
$$\Phi_i(x) \to \Phi'_i(x) = \Phi_i(x) - |\xi_i|^{1/2}m_i\lambda_i(x),$$

for $i = X, 1, 2$ and $\xi_X = 1$. Choosing the unitary gauge, i.e., $\Phi'_i = 0$ for each symmetry transformation we rediscover our initial action.

Higgs mechanism generation of the non-minimal couplings

$$S_{DM} = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{m_X^2}{2}g^{\mu\nu}X_\mu X_\nu + -\frac{\xi_1}{2}g^{\mu\nu}RX_\mu X_\nu + \frac{\xi_2}{2}R^{\mu\nu}X_\mu X_\nu\right\},$$

where $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ with $Z_2 : X_\mu \to -X_\mu$.

$$S_{DM} = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + |D_\mu\Phi|^2 -\frac{\xi_1}{\Lambda^2}R|D_\mu\Phi|^2 + \frac{\xi_2}{\Lambda^2}R^{\mu\nu}(D_\mu\Phi)^\star(D_\nu\Phi).\right\}$$

for $D_\mu\Phi \equiv (\partial_\mu - ig_X X_\mu)\Phi$ and $\Lambda = g_X v_X = m_X$.