SmeftFR v3 – Feynman rules generator for the Standard Model Effective Field Theory

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Abstract

We present version 3 of SmeftFR, a Mathematica package designed to generate the Feynman rules for the Standard Model Effective Field Theory (SMEFT) including the complete set of gauge invariant operators up to dimension-6 and the complete set of bosonic operators of dimension-8. Feynman rules are generated with the use of FeynRules package, directly in the physical (mass eigenstates) basis for all fields. The complete set of interaction vertices can be derived, including all or any chosen subset of SMEFT operators. As an option, the user can also choose preferred gauge fixing, generating Feynman rules in unitary or R_ξ-gauges. The novel feature in version-3 of SmeftFR is its ability to calculate SMEFT interactions consistently up to dimension-8 in EFT expansion (including quadratic dimension-6 terms) and express the vertices directly in terms of user-defined set of input-parameters. The derived Lagrangian in the mass basis can be exported in various formats supported by FeynRules, such as UFO, FeynArts, *etc.* Initialisation of numerical values of Wilson coefficients of higher dimension operators is interfaced to WCxf format. The package also includes a dedicated Latex generator allowing to print the result in clear human-readable form. The SmeftFR v3 is publicly available at www.fuw.edu.pl/smeft.

Keywords: Standard Model Effective Field Theory, Feynman rules, unitary and R_ξ-gauges

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PROGRAM SUMMARY

Manuscript Title:
`SmeftFR` v3 – Feynman rules generator for the Standard Model Effective Field Theory
Authors: A. Dedes, J. Rosiek, M. Ryczkowski, K. Suxho, L. Trifyllis
Program Title: `SmeftFR` v3.0
Journal Reference:
Catalogue identifier:
Licensing provisions: None
Programming language: Mathematica 12.1 or later (earlier versions were reported to have problems running this code)
Computer: any running Mathematica
Operating system: any running Mathematica
RAM: allocated dynamically by Mathematica, at least 4GB total RAM suggested
Number of processors used: allocated dynamically by Mathematica
Supplementary material: None
Keywords: Standard Model Effective Field Theory, Feynman rules, unitary and R_ξ gauges
Classification:
11.1 General, High Energy Physics and Computing,
4.4 Feynman diagrams,
5 Computer Algebra.
External routines/libraries: Wolfram Mathematica program
Subprograms used: `FeynRules v2.3.49` or later package
Nature of problem:
Automatised generation of Feynman rules in physical (mass) basis for the Standard Model Effective Field Theory with user defined operator subset, gauge fixing and input-parameters scheme selection.
Solution method:
Expansion of `SmeftFR` v2 Mathematica package [1]: implementation of the results of Ref. [2] in the `FeynRules` package [3], including dynamic "model files" generation.
Restrictions: None
Unusual features: None
Additional comments: None
Running time: depending on control variable settings, from few minutes for a selected subset of few SMEFT operators and Feynman rules generation up to several hours for generating UFO output for large operator sets (using Mathematica 13.2 running on a personal computer)

References

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[3] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, "FeynRules 2.0 - A complete toolbox for tree-level phenomenology," Comput. Phys. Commun. **185**, 2250 (2014).

1. Introduction

The Standard Model Effective Field Theory (SMEFT) [1–3] is a useful tool in parameterizing phenomena beyond the, successful so far, Standard Model (SM) [4–6] predictions that may appear in current and/or future particle experiments. The SMEFT Lagrangian is given by

$$\mathcal{L}_{\text{SMEFT}} \;=\; \mathcal{L}_{\text{SM}} \;+\; \sum_i \frac{C_i\,\mathcal{O}_i}{\Lambda^{d_i-4}}\;, \tag{1.1}$$

where scale Λ is the cut-off scale of the EFT (i.e., the mass of the lightest heavy particle decoupled from the underlying theory), $\mathcal{O}_i$ is a set of d_i-dimensional, SM gauge group invariant, operators, and C_i are the associated Wilson coefficients (WCs). For one fermion generation including Hermitian conjugation, we have for $d = 5$ two independent operators e.g. $i = 2$, for $d = 6$ we have $i = 84$, for $d = 7$ we have $i = 30$, for $d = 8$ we have $i = 993$, and so on and so forth [7]. When expanding in flavour, the actual number of operators explodes from few to several thousand of operators and interaction vertices. This proliferation of vertices must be included in matrix element calculators when mapping the WCs to experimental data. This is the scope of this article: to describe the code `SmeftFR` v3.0 which consistently provides the Feynman Rules for dimension-6 and the bosonic part of dimension-8 operators for further symbolic or numerical manipulations.

Admittedly, SMEFT is a hugely complicated model. Including all possible CP-, flavour-, baryon-, and lepton-number violating interactions at dimension-6 level, it already contains 2499 free parameters in a non-redundant basis, such as the Warsaw basis [8]. In addition, recent experimental and theoretical progress of high energy processes at LHC involving vector boson scattering requires subsets of dimension-8 operators [9, 10], in particular the bosonic ones, making the structure of possible interactions even more involved. Due to large number and complicated structure of new terms in the Lagrangian, theoretical calculations of physical processes within the SMEFT can be very challenging — it is enough to notice that the number of primary vertices when SMEFT is quantized in R_ξ-gauges and in "Warsaw mass" basis, printed for the first time in ref. [11], is almost 400 without counting the hermitian conjugates.

As a result, it is important to develop technical methods and tools facilitating such calculations, starting from developing the universal set of the Feynman rules for propagators and vertices for physical fields, after Spontaneous Symmetry Breaking (SSB) of the full effective theory in the most commonly studied, Warsaw basis of operators [8]. The initial version of the relevant package, `SmeftFR` v1.0, was announced and briefly described for the first time in Appendix B of ref. [11]. The `SmeftFR` code was further developed and supplied with many new options capabilities and published as `SmeftFR` v2.0 in [12]. In this paper we present `SmeftFR` v3.0, a *Mathematica* symbolic language package generating Feynman rules in several formats, based on the formulae developed in ref. [11]. The most important new capability implemented in the code, comparing to version 2, allows for performing consistent calculations up to dimension-8 operators in EFT expansion, including also expressing the Feynman rules directly in terms of any user-defined set of input parameters. We summarise here the main features of `SmeftFR` code, noting in particular advances introduced in its 3rd version (v3):

- `SmeftFR` is written as an overlay to `FeynRules` package [13, 14], used as the engine to generate Feynman rules.

- `SmeftFR` v3 is able to generate interactions in the most general form of the SMEFT Lagrangian up to dimension-6 order in Warsaw basis [8], without any restrictions on the structure of flavour violating terms and on CP-, lepton- or baryon-number conservation. In addition, it also contains all *bosonic* operators of dimension-8 order, in the basis defined in ref. [9].

- Feynman rules are expressed in terms of physical SM fields and canonically normalised Goldstone and ghost fields. Expressions for interaction vertices are analytically expanded in powers of inverse New Physics scale $1/\Lambda$. The novel feature implemented in `SmeftFR` v3 is the consistent inclusion of all terms up to maximal dimension-8, including both terms quadratic in Wilson coefficients of dimension-6 and linear contributions from Wilson coefficients of dimension-8. Terms of order higher than $d = 8$ are consistently truncated.

- Another important novel feature of `SmeftFR` v3 is the possibility of expressing Feynman rules directly in terms of a predefined set of input parameters (usually chosen to be observables directly measurable in experiments). This allows for consistent calculation of processes in SMEFT without the complicated and error-prone procedure of using "intermediate" set of Lagrangian parameters and later re-expressing them in terms of preferred input quantities.

- `SmeftFR` v3 allows for choosing *any* set of input parameters, assuming that the user provides appropriate routines relating them to "standard" SM Lagrangian parameters (defined later in Sec. 3) to a required (maximum 8th) order of SMEFT expansion. Two most frequently used input schemes in the electroweak sector, (G_F, M_Z, M_W, M_H) and $(\alpha_{em}, M_Z, M_W, M_H)$ are predefined in current version, including all terms up to dimension-8. In both cases, the strong coupling constant and all quark and lepton masses are also inputs. In addition, `SmeftFR` v3 also includes a predefined input scheme for the CKM matrix adopted from ref. [15]. For the neutrino mixing matrix we use as input the standard PMNS matrix not (as yet) corrected by SMEFT.

- Including the full set of SMEFT parameters in model files for `FeynRules` may lead to very slow computations. `SmeftFR` can generate `FeynRules` model files dynamically, including only the user defined subset of higher dimension operators. It significantly speeds up the calculations and produces a simpler final result, containing only the Wilson coefficients relevant to the process that she/he has chosen to analyse. It is worth noting that optimisations included in `SmeftFR` v3 sped it up comparing to `SmeftFR` v2 by approximately an order of magnitude for a comparable subset of chosen operators of dimension-6 and calculations done up to $1/\Lambda^2$ accuracy (maximally achievable in `SmeftFR` v2).

- Feynman rules can be generated in the unitary or in linear R_ξ-gauges by exploiting four different gauge-fixing parameters $\xi_\gamma, \xi_Z, \xi_W, \xi_G$ for thorough amplitude checks. In the latter case also all relevant ghost and Goldstone vertices are obtained. This procedure is described in detail in ref. [11] and implemented already in `SmeftFR` v2 [12].

- Feynman rules are calculated first in *Mathematica*/`FeynRules` format. They can be further exported in other formats: `UFO` [16] (importable to Monte-Carlo generators like

MadGraph5_aMC@NLO 5 [17], Sherpa [18], CalcHEP [19], Whizard [20, 21]), FeynArts [22] which generates inputs for loop amplitude calculators like FeynCalc [23], or FormCalc [24], and other output types supported by FeynRules .

- SmeftFR provides a dedicated Latex generator, allowing to display vertices and analytical expressions for Feynman rules in clear human readable form, best suited for hand-made calculations.
- SmeftFR is interfaced to the WCxf format [25] of Wilson coefficients. Numerical values of SMEFT parameters in model files can be read from WCxf JSON-type input produced by other computer codes written for SMEFT. Alternatively, SmeftFR can translate FeynRules model files to the WCxf format.
- Further package options allow to treat neutrino fields as massless Weyl or (in the case of non-vanishing dimension-5 operator) massive Majorana fermions, to correct signs in 4-fermion interactions not yet fully supported by FeynRules and to perform some additional operations as described later in this manual.

It has also been made and tested to be compatible with many other publicly available high-energy physics related computer codes accepting standardised input and output data formats.

Feynman rules derived in ref. [11] using the SmeftFR package have been used successfully in many articles, including refs. [26–60], and have passed certain non-trivial tests, such as gauge-fixing parameter independence of the S-matrix elements, validity of Ward identities, cancellation of infinities in loop calculations, *etc.*

We note, here, that there is a growing number of publicly available codes performing computations related to SMEFT [61]. These include, Wilson [62], Flavio [63], DSixTools [64, 65], RGESolver [66], MatchingTools [67], CoDEx [68], HighPT [69], STream [70], SuperTracer [71], Matchmakereft [72], Matchete [73], which are codes for running and matching Wilson coefficients and FeynOnium [74] for automatic calculations in non-relativistic EFTs. Packages mostly relevant to the purposes of SmeftFR are SMEFTsim [75, 76], Dim6Top [77] and SMEFT@NLO [78] which are all codes for calculating physical observables in SMEFT. To a degree, these codes (especially the ones supporting WCxf format) can be used in conjunction with SmeftFR. For example, some of them can provide the numerical input for Wilson coefficients of higher dimensional operators at scale Λ, while others, the running of these coefficients from that scale down to the EW one. Alternatively, Feynman rules evaluated by SmeftFR can be used with Monte-Carlo event generators to test the predictions of other codes.

The rest of the paper is organised as follows. In Sec. 2, we define the notation and conventions of the SMEFT Lagrangian and the field normalisations used in transition to mass basis. In Sec. 3 and Appendix A, we describe the input schemes, i.e. the user-defined choices of observables which can be used to parametrize SMEFT interactions and give examples of the corresponding output of the code. In Sec. 4, we present the code's algorithmic structure and installation procedure. Sec. 5 is the main part of the paper, illustrating in detail how to derive the set of SMEFT vertices in mass basis starting from $d = 6$ operators in Warsaw basis [8] and $d = 8$ bosonic operators in basis of ref. [9] (all operators used by SmeftFR v3 are collected for completeness in Appendix B). A sample program with `SmeftFR` v3 commands, generating Feynman rules in various formats, is given in Sec. 6. We conclude in Sec. 8.

2. SMEFT Lagrangian in gauge and mass basis

The classification of higher order operators in SMEFT is done in terms of fields in electroweak basis, before Spontaneous Symmetry Breaking (SSB). For the dimension-5 and -6 operators, `SmeftFR` uses the so-called "Warsaw basis" [8] as a starting point to calculate physical states in SMEFT and their interactions (for the specification of Warsaw basis, see ref. [8], in particular eq. (3.1) defining the $d = 5$ Weinberg operator $Q^{(5)}_{\nu\nu}$ and Tables 2 and 3 containing the full list of $d = 6$ operators). For the dimension-8 operators, we include all operators containing bosonic fields only, as listed in Tables 2 and 3 of ref. [9] with an exception of two operators. The definitions and the list of all operators used by `SmeftFR` v3 is described in Appendix B, and Tables B.1, B.2, B.3, and B.4.

We decided to neglect $d = 7$ (which always contain fermionic fields) and fermionic $d = 8$ operators, both for theoretical and practical purposes. Dimension-7 operators are all lepton or baryon number violating and strongly constrained by many, related, experiments. In most BSM models, dimension-8 operators are also strongly suppressed and can lead to substantial measurable effects only when their contributions are enhanced, which typically (as can be justified on dimensional ground) happens at high energies. Such effects could be in particular investigated in experimental searches that involve Vector Boson Scattering at the LHC (see e.g. [45, 79–82]), and therefore, including bosonic operators is particularly important for such contemporary analyses. Furthermore, fermionic $d = 8$ operators, either pure or mixed with other fields, may be equally important for collider studies. Chosen higher order fermionic operators can also be loaded in `SmeftFR` , however, as we will discuss in Section 7, at present it requires introducing certain modifications and thus some expertise in the code structure.

The SMEFT Lagrangian which we use is the sum of the dimension-4 terms and operators of order up to dimension-8 (the latter only in the bosonic sector):

$$\mathcal{L} = \mathcal{L}^{(4)}_{\rm SM} + \frac{1}{\Lambda} C^{\nu\nu} Q^{(5)}_{\nu\nu} + \frac{1}{\Lambda^2} \sum_{boson,fermion} C^{(6)}_{(b,f)} Q^{(6)}_{(b,f)} + \frac{1}{\Lambda^4} \sum_{boson} C^{(8)}_{b} Q^{(8)}_{b} \,. \tag{2.1}$$

Physical fields in SMEFT are obtained after SSB. In the gauge and Higgs sectors, physical and Goldstone fields $(h, G^0, G^\pm, W^\pm_\mu, Z^0_\mu, A_\mu)$ are related to initial (Warsaw basis) fields $(\varphi, W^i_\mu, B_\mu, G^A_\mu)$ by field normalisation constants:[1]

$$\begin{aligned}
\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} &= \begin{pmatrix} Z_{G^+}^{-1} G^+ \\ \frac{1}{\sqrt{2}}(v + Z_h^{-1} h + i Z_{G^0}^{-1} G^0) \end{pmatrix} , \\
\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} &= Z_{\gamma Z} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} ,
\end{aligned}$$

[1] Note the notation difference with ref. [11]: Quantities Z_W and Z_G defined in eq. 2.2 are denoted as their inverses, Z_W^{-1} and Z_G^{-1}, in ref. [11].

Constant	Variable	Constant	Variable
Z_{g_s}	`gsnorm`	Z_G	`Gnorm`
Z_g	`gwnorm`	Z_W	`Wnorm`
$Z_{g'}$	`g1norm`	$Z_{\gamma Z}^{ij}$	`AZnorm[i,j]`
Z_h	`Hnorm`	Z_{G^0}	`G0norm`
Z_{G^+}	`GPnorm`		

Table 1: Names of normalisation constants and corresponding internal `SmeftFR` variables.

$$\begin{aligned} W_\mu^1 &= \frac{Z_W}{\sqrt{2}} (W_\mu^+ + W_\mu^-) \,, \\ W_\mu^2 &= \frac{i Z_W}{\sqrt{2}} (W_\mu^+ - W_\mu^-) \,, \\ G_\mu^A &= Z_G \, g_\mu^A \,. \end{aligned} \tag{2.2}$$

In addition, we define the effective gauge couplings, chosen to preserve the natural form of covariant derivative:

$$g = Z_g \bar{g} \qquad g' = Z_{g'} \bar{g}' \qquad g_s = Z_{g_s} \bar{g}_s \,. \tag{2.3}$$

Up to $d = 8$, the normalisation constants multiplying the gauge couplings read as:

$$Z_g = \left(1 - \frac{2v^2}{\Lambda^2} C_{\varphi W} - \frac{v^4}{\Lambda^4} C_{W2\varphi 4n1}\right)^{1/2} \,, \tag{2.4}$$

$$Z_{g'} = \left(1 - \frac{2v^2}{\Lambda^2} C_{\varphi B} - \frac{v^4}{\Lambda^4} C_{B2\varphi 4n1}\right)^{1/2} \,, \tag{2.5}$$

$$Z_{g_s} = \left(1 - \frac{2v^2}{\Lambda^2} C_{\varphi G} - \frac{v^4}{\Lambda^4} C_{G2\varphi 4n1}\right)^{1/2} \,, \tag{2.6}$$

where relevant operators are defined in [8, 9] and formally all expressions have to be expanded to the order $\frac{v^4}{\Lambda^4}$.

The above field normalisation constants Z_X, the corrected Higgs field vev, v, and the gauge and Higgs boson masses, M_Z, M_W and M_h, are not encoded as fixed analytical expressions but calculated by `SmeftFR` using the condition that bilinear part of the Lagrangian must have canonical form in the mass eigenstates basis. In this way, all relations automatically contain only the subset of non-vanishing SMEFT Wilson coefficients chosen by the user, as described in Sec. 5. The analytical expressions for the normalisation constants for a chosen set of higher dimension operators after running `SmeftFR` initialisation procedure are stored in variables listed in Table 1 (as discussed later in Sec. 3, expressions for the SM parameters in terms of user-defined input quantities are also available, see Table 2). One should note that *at any order* in SMEFT, $SU(2)$ and $SU(3)$ gauge field and gauge normalisation constants are related, $Z_W = Z_g^{-1}$, $Z_G = Z_{g_s}^{-1}$.

It is also easy to eventually further expand the program in future by adding even higher than dimension-8 operators, as the routine diagonalizing the field bilinears does not depend on their particular dependence on Wilson coefficients of higher dimension operators until the very final stage where such dependence is substituted and further expanded in $1/\Lambda$ powers.

The basis in the fermion sector is not fixed by the structure of gauge interactions and allows for unitary rotation freedom in the flavour space:

$$\psi'_X = U_{\psi_X}\,\psi_X\ , \tag{2.7}$$

with $\psi = \nu, e, u, d$ and $X = L, R$. We choose the rotations such that ψ_X eigenstates correspond to real and non-negative eigenvalues of 3×3 fermion mass matrices:

$$\begin{aligned} M'_\nu &= -v^2 C'^{\nu\nu}\ , & M'_e &= \tfrac{v}{\sqrt{2}}\left(\Gamma_e - \tfrac{v^2}{2}C'^{e\varphi}\right), \\ M'_u &= \tfrac{v}{\sqrt{2}}\left(\Gamma_u - \tfrac{v^2}{2}C'^{u\varphi}\right), & M'_d &= \tfrac{v}{\sqrt{2}}\left(\Gamma_d - \tfrac{v^2}{2}C'^{d\varphi}\right). \end{aligned} \tag{2.8}$$

The fermion flavour rotations can be adsorbed in redefinitions of Wilson coefficients, as a result leaving CKM and PMNS matrices (denoted in `SmeftFR` as K and U respectively) multiplying them. The complete list of redefinitions of flavour-dependent Wilson coefficients is given in Table 4 of ref. [11]. After rotations, they are defined in so called "Warsaw mass" basis (as also described in WCxf standard [25]). `SmeftFR` assumes that the numerical values of Wilson coefficients of $d = 6$ fermionic operators (see Table B.1) are given in this particular basis.

In summary, Feynman rules generated by the `SmeftFR` code describe interactions of SMEFT physical (mass eigenstates) fields, with numerical values of Wilson coefficients defined in the "Warsaw mass" basis of ref. [11] extended with bosonic subset of dimension-8 operators in the basis defined in ref. [9].

It is also important to stress that in the general case of lepton number flavour violation, with the non-vanishing dimension-5 Weinberg operator $Q^{(5)}_{\nu\nu}$, neutrinos are massive Majorana spinors, whereas under the assumption of L-conservation they can be regarded as massless Weyl spinors. As described in the Sec. 5.1, `SmeftFR` is capable to generate Feynman rules for neutrino interactions in both cases, depending on the choice of initial options[2]. One should note that although for pure $V-A$ neutrino-gauge boson interactions in the SM the predictions for physical observables almost never depend on the character of neutrino fields (Dirac or Majorana), this is no longer true in case of non-standard neutrino couplings generated by higher dimension operators. Detailed discussion of such issues, with relevant examples of different predictions for 2- and 3-body decays involving pair of Dirac or Majorana neutrinos in the final state, can be found in refs. [86, 87].

3. Parametrization of the SMEFT interactions

3.1. SMEFT input parameter selection

The standard way of parameterizing the SMEFT Lagrangian is to use the natural set of couplings defining the dimension-4 renormalizable interactions (i.e. the SM Lagrangian)

[2]One should remember that treating neutrinos as Majorana particles requires special set of rules for propagators, vertices, and diagram combinatorics. We follow here the treatment described in refs. [11, 83–85].

supplied with the Wilson coefficients of the higher order operators. The commonly used set of quantities parameterizing the $d = 4$ part of Lagrangian is:

$$\begin{array}{rl} \bar{g}, \bar{g}', \bar{g}_s & SU(2), U(1), SU(3) \text{ gauge couplings} \\ v, \lambda & \text{Higgs boson mass and quartic coupling} \\ m_q & \text{quark masses, } q = u, c, t, d, s, b \\ K & \text{CKM quark mixing matrix} \\ m_\ell, m_{\nu_\ell} & \text{charged lepton and neutrino masses, } \ell = e, \mu, \tau \\ U & \text{PMNS lepton mixing matrix} \end{array} \tag{3.1}$$

In the list above we assume that gauge couplings $\bar{g}, \bar{g}', \bar{g}_s$ are already redefined as in eq. (2.3) and v is the minimum of the full Higgs boson potential, including the higher order operators.

SMEFT Feynman rules evaluated by `SmeftFR` v3 can be expressed in terms of such set of parameters and WCs of higher dimension operators. We further called it to be the "default" parametrization set, selected using `Option` $\rightarrow$ `''smeft''` in various routines of the code, as described in Sec. 4. Expressing observables calculated in SMEFT in terms of "default" parameter gives a natural extension of the corresponding formulae in SM, as the latter can be immediately obtained by setting all WCs to zero. However, some parameters in eq. (3.2), namely gauge and Higgs couplings, K and U mixing matrices (also particle masses if they are not chosen to be physical pole masses) are not directly measurable quantities. Their numerical values in SMEFT have to be derived by choosing an appropriate "input parameter scheme", i.e. set of observables $O_1, \ldots, O_n$, and expressing them in terms of such input parameters and WCs:

$$\begin{array}{rcl} \bar{g} & = & \bar{g}(O_1, \ldots, O_n, C_i) \; , \\ \bar{g}' & = & \bar{g}'(O_1, \ldots, O_n, C_i) \; , \\ & \ldots & . \end{array} \tag{3.2}$$

Such a procedure leads to additional complications in calculating processes within SMEFT. All physical quantities have to be consistently calculated to a given order of $1/\Lambda$ expansion in order to keep the result gauge invariant. Therefore, any observable, $\mathcal{A}$, calculated in terms of "default" parameters of eq. (3.2) has to be re-expanded to a given EFT order after expressing in terms of input parameters:

$$\begin{aligned} \mathcal{A} &= \mathcal{A}_4(\bar{g}, \bar{g}', \ldots) + \frac{1}{\Lambda^2} \mathcal{A}_6^i(\bar{g}, \bar{g}', \ldots) C_6^i \\ &+ \frac{1}{\Lambda^4} \left(\mathcal{A}_8^{1ij}(\bar{g}, \bar{g}', \ldots) C_6^i C_6^j + \mathcal{A}_8^{2i}(\bar{g}, \bar{g}', \ldots) C_8^i \right) + \ldots \\ &= \mathcal{A'}_4(O_1, O_2, \ldots) + \frac{1}{\Lambda^2} \mathcal{A'}_6^i(O_1, O_2, \ldots) C_6^i \\ &+ \frac{1}{\Lambda^4} \left(\mathcal{A'}_8^{1ij}(O_1, O_2, \ldots) C_6^i C_6^j + \mathcal{A'}_8^{2i}(O_1, O_2, \ldots) C_8^i \right) + \ldots \end{aligned} \tag{3.3}$$

where for simplicity we neglected odd powers in $1/\Lambda$ expansion as they are always lepton or baryon number violating and strongly suppressed.

Re-expressing SMEFT amplitudes and re-expanding them in $1/\Lambda$ powers can be technically tedious and error-prone, especially at higher EFT orders. Therefore, it is useful to have SMEFT interaction vertices expressed from the very beginning directly in terms of a set of measurable physical observables. Calculations done in terms of such Feynman rules can be simply truncated at required EFT order, without the need of re-parametrization. `SmeftFR` v3 provides such capability of evaluating the SMEFT Lagrangian and interaction vertices directly in terms of *any* user defined set of input parameters.

3.2. User-defined input parameters

`SmeftFR` v3 allows users to choose their own preferred set of input parameters, providing they are defined in the correct format and related to the "default" parameters set defined in eq. (3.2). The user-defined input parameters in `SmeftFR` should fulfil the following conditions:

- they are assumed to be measurable physical observables or other quantities which do not depend on the SMEFT parameters, in particular on WCs of higher dimension operators.
- they should be real scalar numbers, i.e. do not carry any flavor or gauge indices. If necessary, indexed arrays of flavor or gauge parameters should be represented by the relevant set of separate scalar entries.
- names of user-defined parameters should not overlap the names of variables already used by the code. `SmeftFR` performs checks for overlapping names of variables and displays if necessary relevant warnings.
- user-defined parameters and relations between them and "default" parameters should be defined in the file `code/smeft_input_scheme.m`.
- the format for defining user input parameters follows the standard format of `FeynRules` model definition files, as illustrated in the example below:

```
SM$InputParameters = {
(* observables used as input parameters in gauge and Higgs sector *)
      alphas == {
           ParameterType -> External,
           Value -> 0.1176,
           InteractionOrder -> {QCD,2},
           TeX -> Subscript[\[Alpha],s],
           Description -> "average alpha_s at MZ scale"
      },
     ...
}
```

  A more detailed example of user input parameter definition can be found in the header of the file `code/smeft_input_scheme.m` supplied with the `SmeftFR` v3 distribution.

- the chosen set of user input parameters must be sufficient to fully define "default" SMEFT parameters in terms of them and WCs of higher dimension operators. After choosing their own input parameters, further referred to as “input schemes”, the users are supposed to provide the corresponding routine with analytical expressions for *all* variables listed in Table 2. The example of such a routine and predefined most-often used SMEFT input scheme are again provided in the file `code/smeft_input_scheme.m` (see routine `SMEFTInputScheme`).

Gauge and Higgs sector		Quark sector		Lepton sector	
UserInput$vev	v	UserInput$MQU	m_u	UserInput$MLE	m_e
UserInput$GW	$\bar{g}$	UserInput$MQC	m_c	UserInput$MLM	m_μ
UserInput$G1	$\bar{g}'$	UserInput$MQT	m_t	UserInput$MLT	m_τ
UserInput$GS	$\bar{g}_s$	UserInput$MQD	m_d	UserInput$MVE	m_{ν_e}
UserInput$hlambda	λ	UserInput$MQS	m_s	UserInput$MVM	m_{ν_μ}
UserInput$MZ	M_Z	UserInput$MQB	m_b	UserInput$MVT	m_{ν_τ}
UserInput$MW	M_W	UserInput$CKM	K	UserInput$PMNS	U
UserInput$MH	M_H				

Table 2: Names of normalisation constants and corresponding internal `SmeftFR` variables.

3.3. Predefined input schemes

Although `SmeftFR` v3 in principle allows defining any set of user-defined input parameters, some input schemes are more natural and technically easier to use than others. In particular, it is almost obligatory to use physical masses of SM particles as part of the input parameter set. Otherwise, if masses are calculated as combinations of other variables and WCs, the latter appear in the particle propagators, making all amplitude calculations and $1/\Lambda$ expansions significantly more difficult. This leaves only $\bar{g}$, $\bar{g}'$, the vev v, and λ in the electroweak sector, $\bar{g}_s$ in the strong sector, CKM matrix K in the quark sector and PMNS matrix U in the lepton sector to be defined in terms of input parameters.

`SmeftFR` v3 provides predefined routines realising the most commonly used SMEFT input schemes which can be selected by calling the `SMEFTInputScheme` routine with relevant options:

- Gauge sector:
  - (G_F, M_Z, M_W, M_H) input scheme or
  - $(\alpha_{em}, M_Z, M_W, M_H)$ input scheme

  where M_Z, M_W, M_H are the physical gauge and Higgs boson masses and G_F is the Fermi constant derived from the muon lifetime.

  In both cases “default” electroweak sector parameters $\bar{g}, \bar{g}', v$ and λ are expressed in terms of input parameters listed above including linear and quadratic corrections from all contributing $d = 6$ operators and linear corrections from only-bosonic $d = 8$ operators.

  Strong coupling $\bar{g}_s$ is defined as $\sqrt{4\pi\alpha_s(M_Z)}$ with some input value of $\alpha_s(M_Z)$ assumed. Currently, `SmeftFR` v3 distribution does not include *any* corrections from higher order operators, leaving it eventually to further modifications by users. It is not an easy task - the experimental value of $\alpha_s(M_Z)$ cited in literature is an average from various types of measurements. The correct derivation of such an average in SMEFT should take into account the fact that different processes used to determine $\alpha_s(M_Z)$ are affected in different ways by the presence of the higher dimension operators, thus the relation of the "averaged" $\alpha_s(M_Z)$ to $\bar{g}_s$ has a complicated dependence on WCs of such operators. To our knowledge, no such analysis exists yet in the literature, providing formulae which could be implemented in the symbolic or numerical codes.

- Quark sector: Quark masses are assumed to be their physical masses - even if such notion is unclear in case of light quarks, their values usually do not affect in substantial way most of practical calculations, so also the exact definitions are not so important in this case. Corrections to CKM matrix K are evaluated using the formulae derived in ref. [15]. They are accurate up to $d = 6$ linear terms.

  One should note that non-vanishing values of some flavor off-diagonal 4-quark operators can lead to numerically very large corrections to CKM elements. If they are larger than 20%, `SmeftFR` v3 displays a relevant warning and does not include corrections to CKM matrix at all. They can be forced to be included independently on how large they appear using the option `CKMInput` $\to$ `"force"` in `SMEFTInitializeModel` routine.

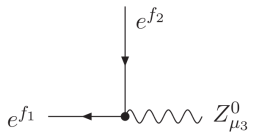
- Lepton sector: Charged lepton masses are assumed to be physical masses. Neutrino masses are calculated as proportional to the WC of $d = 5$ Weinberg operator, $m_{\nu_i} = v^2|C^i_{\nu\nu}|$. The PMNS matrix is currently evaluated from measured neutrino mixing angles without including corrections from higher order operators, again leaving it to eventual future modifications by users.

In the predefined input scheme routines in the gauge sector, all re-parametrizations are done analytically. Analytical formulae for corrections to K matrix element are lengthy and complicated, leading to very long and hardly readable expressions for interaction vertices and as result also transition amplitudes. Therefore, currently, corrections to CKM matrix elements from the $d = 6$ operators are in `SmeftFR` v3 evaluated numerically and added to SM central values.

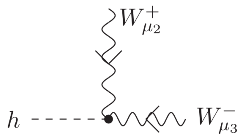
3.4. Output parametrization

Following the options described above, `SmeftFR` v3 can calculate the interaction vertices in mass basis parametrized in three (user-selectable) forms:

1. The "unexpanded" (selected as option `Expansion` $\to$ `"none"` in relevant routines as described in Sec. 5) parametrization. Interaction vertices are given in terms of "default" parameters, WCs and Z_X normalisation constants without expressing the latter explicitly in terms of "default" or "user-defined" parameters. Such output is compact and fast to produce. Also, it is the most universal one - adding additional higher order operators (like fermionic $d = 8$ operators or even higher EFT orders), apart from directly appearing new vertices, can be easily accommodated by adding new contributions to expressions for Z_X. However, in such form, consistent expansion to a given EFT order is hidden and



$$
\begin{aligned}
&+ \quad \frac{i}{2}\delta_{f_1 f_2}\left(\bar{g}' Z_{g'} Z^{21}_{\gamma Z}\left(\gamma^{\mu_3} P_L + 2\gamma^{\mu_3} P_R\right) + \bar{g} Z_g Z^{11}_{\gamma Z}\gamma^{\mu_3} P_L\right)\\
&- \quad \sqrt{2} v Z^{21}_{\gamma Z} p_3^{\nu}\left(C^{eB*}_{f_2 f_1}\sigma^{\mu_3\nu} P_L + C^{eB}_{f_1 f_2}\sigma^{\mu_3\nu} P_R\right)\\
&+ \quad \sqrt{2} v Z^{11}_{\gamma Z} p_3^{\nu}\left(C^{eW*}_{f_2 f_1}\sigma^{\mu_3\nu} P_L + C^{eW}_{f_1 f_2}\sigma^{\mu_3\nu} P_R\right)\\
&+ \quad \frac{iv^2}{2}\gamma^{\mu_3} P_R\left(\bar{g} Z_g Z^{11}_{\gamma Z} - \bar{g}' Z_{g'} Z^{21}_{\gamma Z}\right) C^{\varphi e}_{f_1 f_2}\\
&+ \quad \frac{iv^2}{2}\gamma^{\mu_3} P_L\left(\bar{g} Z_g Z^{11}_{\gamma Z} - \bar{g}' Z_{g'} Z^{21}_{\gamma Z}\right) C^{\varphi l1}_{f_1 f_2}\\
&+ \quad \frac{iv^2}{2}\gamma^{\mu_3} P_L\left(\bar{g} Z_g Z^{11}_{\gamma Z} - \bar{g}' Z_{g'} Z^{21}_{\gamma Z}\right) C^{\varphi l3}_{f_1 f_2}
\end{aligned}
$$



$$
+\frac{i\bar{g}^2 v}{2Z_h}\eta_{\mu_2\mu_3} + \frac{4iv}{Z_g^2 Z_h}\left(p_2^{\mu_3} p_3^{\mu_2} - p_2\cdot p_3\eta_{\mu_2\mu_3}\right) C^{\varphi W}
$$

Figure 1: $Z\ell^+\ell^-$ and hW^+W^- vertices before expansion of Z_X couplings (including a sample list of operators up to maximal dimension-6). For simplicity in displaying every Feynman rule, the $1/\Lambda^2$-factor accompanying every $d = 6$ Wilson Coefficient is omitted e.g. $C^{\varphi W} \to C^{\varphi W}/\Lambda^2$.

can be done only after substituting explicit expressions for Z_X. Sample vertices in such parametrization are displayed in Fig. 1.

2. The "default" (chosen by the option `Expansion -> "smeft"`) parametrization. Interaction vertices are given in terms of "default" parameters and WCs, with shifts of SM fields and couplings expanded accordingly. The result is truncated to user-selectable EFT order ($d = 4$, 6 or 8). Sample vertices in such parametrization are displayed in Fig. 2.
3. The "user" (chosen by the option `Expansion -> "user"`) parametrization. Interaction vertices are given directly in terms of user-defined input parameters and WCs, again with shifts of SM fields and couplings expanded accordingly. The result is truncated to user-selectable EFT order ($d = 4$, 6 or 8). Sample vertices for the (G_F, M_Z, M_W, M_h) input scheme in the electroweak sector (see discussion in Sec. 3.3) are displayed in Fig. 3.

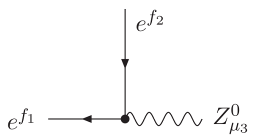
As described in more details in the next Section, the form of the output can be selected by choosing various code options.

4. `SmeftFR` installation and code structure

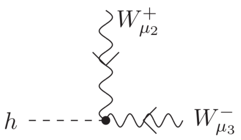
4.1. Installation

`SmeftFR` package works using the `FeynRules` system, so both need to be properly installed first. A recent version and installation instructions for the `FeynRules` package can be downloaded from the address:

`https://feynrules.irmp.ucl.ac.be`



$$
\begin{aligned}
&- \frac{i}{2\sqrt{\bar{g}'^2+\bar{g}^2}}\delta_{f_1f_2}\left(\left(\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L+2\bar{g}'^2\gamma^{\mu_3}P_R\right)\\
&+ \frac{i\bar{g}'\bar{g}v^2}{2\left(\bar{g}'^2+\bar{g}^2\right)^{3/2}}\delta_{f_1f_2}\left(\left(\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L-2\bar{g}^2\gamma^{\mu_3}P_R\right)C^{\varphi WB}\\
&+ \frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}'^2+\bar{g}^2}}p_3^{\nu}\left(C^{eB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+ \frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}'^2+\bar{g}^2}}p_3^{\nu}\left(C^{eW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+ \frac{1}{2}iv^2\sqrt{\bar{g}'^2+\bar{g}^2}C^{\varphi e}_{f_1f_2}\gamma^{\mu_3}P_R+\frac{1}{2}iv^2\sqrt{\bar{g}'^2+\bar{g}^2}C^{\varphi l1}_{f_1f_2}\gamma^{\mu_3}P_L\\
&+ \frac{1}{2}iv^2\sqrt{\bar{g}'^2+\bar{g}^2}C^{\varphi l3}_{f_1f_2}\gamma^{\mu_3}P_L
\end{aligned}
$$



$$
\begin{aligned}
&+ \frac{1}{2}i\bar{g}^2v\eta_{\mu_2\mu_3}+\frac{1}{2}i\bar{g}^2v^3\eta_{\mu_2\mu_3}C^{\varphi\Box}-\frac{1}{8}i\bar{g}^2v^3\eta_{\mu_2\mu_3}C^{\varphi D}\\
&+ 4ivC^{\varphi W}\left(p_2^{\mu_3}p_3^{\mu_2}-p_2\cdot p_3\eta_{\mu_2\mu_3}\right)
\end{aligned}
$$

Figure 2: Same as in Fig. 1 but in default ($\bar{g}'$, $\bar{g}$, v) parametrization scheme (the Z_X couplings are expanded up to maximal dimension-6 terms).

`SmeftFR` v3 has been tested with `FeynRules` version 2.3.49. It should be used with *Mathematica* version 12.1 or later, as also the newest `FeynRules` version was modified to be compatible with *Mathematica* upgrades.

Standard `FeynRules` installation assumes that the new models' description is put into `Models` sub-directory of its main tree. We follow this convention, so that the `SmeftFR` file archive should be unpacked into

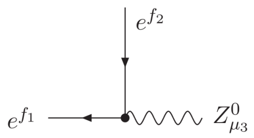
`Models/SMEFT_N_NN`

catalogue, where `N_NN` denotes the package version (currently version `3_00`). After installation, `Models/SMEFT_N_NN` contains the following files and sub-directories listed in Table 3.

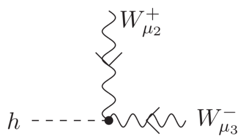
Before running the package, one needs to set properly the main `FeynRules` installation directory, defining the `$FeynRulesPath` variable at the beginning of `smeft_fr_init.m` and `smeft_fr_interfaces.m` files. For non-standard installations (not advised!), also the variable `SMEFT$Path` has to be updated accordingly.

4.2. Code structure

The most general version of SMEFT, including all possible flavour violating couplings, is very complicated. Symbolic operations on the full SMEFT Lagrangian, including the complete set of dimension-5 and-6 operators and bosonic dimension-8 operators, with numerical values of all Wilson coefficients assigned, are time-consuming and can take hours or even days on a standard personal computer. For most of the physical applications it is sufficient to derive



$$
\begin{aligned}
& - \frac{i2^{1/4}\sqrt{G_F}}{M_Z}\delta_{f_1f_2}\left(\left(M_Z^2-2M_W^2\right)\gamma^{\mu_3}P_L+2\left(M_Z^2-M_W^2\right)\gamma^{\mu_3}P_R\right)\\
& + \frac{i2^{3/4}M_W\sqrt{M_Z^2-M_W^2}}{M_Z\sqrt{G_F}}\delta_{f_1f_2}C^{\varphi WB}\gamma^{\mu_3}\\
& + \frac{2^{1/4}\sqrt{M_Z^2-M_W^2}}{\sqrt{G_F}M_Z}p_3^\nu\left(C^{eB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
& + \frac{2^{1/4}M_W}{\sqrt{G_F}M_Z}p_3^\nu\left(C^{eW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
& + \frac{i\,\delta_{f_1f_2}}{2^{9/4}\sqrt{G_F}M_Z}C^{\varphi D}\left(\left(2M_W^2+M_Z^2\right)\gamma^{\mu_3}P_L+2\left(M_W^2+M_Z^2\right)\gamma^{\mu_3}P_R\right)\\
& + \frac{iM_Z}{2^{1/4}\sqrt{G_F}}C^{\varphi e}_{f_1f_2}\gamma^{\mu_3}P_R+\frac{iM_Z}{2^{1/4}\sqrt{G_F}}C^{\varphi l1}_{f_1f_2}\gamma^{\mu_3}P_L+\frac{iM_Z}{2^{1/4}\sqrt{G_F}}C^{\varphi l3}_{f_1f_2}\gamma^{\mu_3}P_L\\
& + \frac{i\,\delta_{f_1f_2}}{2^{9/4}\sqrt{G_F}M_Z}C^{ll}_{2112}\left(\left(M_Z^2-2M_W^2\right)\gamma^{\mu_3}P_L+2\left(M_Z^2-M_W^2\right)\gamma^{\mu_3}P_R\right)\\
& + \frac{i\,\delta_{f_1f_2}}{2^{9/4}\sqrt{G_F}M_Z}\left(C^{\varphi l3}_{11}+C^{\varphi l3}_{22}\right)\left(\left(2M_W^2-M_Z^2\right)\gamma^{\mu_3}P_L+2\left(M_W^2-M_Z^2\right)\gamma^{\mu_3}P_R\right)
\end{aligned}
$$



$$
\begin{aligned}
& + i2^{3/4}\sqrt{G_F}M_W^2\eta_{\mu_2\mu_3}+\frac{i2^{3/4}M_W^2}{\sqrt{G_F}}\eta_{\mu_2\mu_3}C^{\varphi\Box}-\frac{iM_W^2}{2^{3/4}\sqrt{G_F}}\eta_{\mu_2\mu_3}C^{\varphi D}\\
& - \frac{iM_W^2}{2^{3/4}\sqrt{G_F}}\eta_{\mu_2\mu_3}C^{ll}_{2112}+\frac{iM_W^2}{2^{3/4}\sqrt{G_F}}\eta_{\mu_2\mu_3}\left(C^{\varphi l3}_{11}+C^{\varphi l3}_{22}\right)\\
& + \frac{i2^{7/4}}{\sqrt{G_F}}C^{\varphi W}\left(p_2^{\mu_3}p_3^{\mu_2}-p_2\cdot p_3\eta_{\mu_2\mu_3}\right)
\end{aligned}
$$

Figure 3: Same as in Fig. 1 but in the (G_F, M_Z, M_W, M_h) input scheme (the Z_X couplings are expanded up to maximal dimension-6 terms).

interactions only for a subset of operators.[3]

To speed up the calculations, `SmeftFR` can evaluate Feynman rules for a chosen subset of operators only, generating dynamically the proper `FeynRules` "model files". The calculations are divided in three stages, as illustrated in the flowchart of Fig. 4.

- First, before initialising the `FeynRules` engine, a routine relating default and user-defined input parameters are executed. Numerical values of parameters depending on WCs of higher order operators are calculated. Then, two `FeynRules` model files for SMEFT (for gauge and mass basis) are dynamically generated, containing all variables required to fully describe interactions in various parametrizations (see Sec. 3.4).

- Next, the SMEFT Lagrangian is initialised in gauge basis and transformed to mass eigenstates basis analytically. At this stage, Z_X normalisation constants are evaluated in terms of both "default" and "user-defined" input parameters, but such explicit expressions are not substituted in interaction vertices. This very significantly speeds up the calculations (approximately by an order of magnitude comparing to `SmeftFR` v2) and produces ex-

[3]Eventually, operators must be selected with care as in general they may mix under renormalisation [88–90].

`SmeftFR-init.nb` `smeft_fr_init.m`	Notebook and equivalent text script generating SMEFT Lagrangian in mass basis and Feynman rules in *Mathematica* format.
`SmeftFR-interfaces.nb` `smeft_fr_interfaces.m`	Notebook and text script with routines for exporting Feynman rules in various formats: WCxf, Latex, UFO and FeynArts.
`SmeftFR_v3.pdf`	package manual in pdf format.
`code`	sub-directory with package code and utilities.
`lagrangian`	sub-directory with expressions for the SM Lagrangian and dimension-5, 6 and 8 operators coded in `FeynRules` format.
`definitions`	sub-directory with templates of SMEFT "model files" and example of numerical input for Wilson coefficients in WCxf format.
`output`	sub-directory with dynamically generated model "parameter files" and output for Feynman rules in various formats, by default *Mathematica*, Latex, UFO and FeynArts are generated.

Table 3: Files and directories included in `SmeftFR` v3.00 package.

pressions that are remarkably compact for such a complicated model. All terms which are explicitly of order in $1/\Lambda$ higher than requested by users (maximum $1/\Lambda^4$) are truncated, but for consistent $1/\Lambda$ expansions such terms must be neglected once more after inserting an explicit expression for Z_X. The resulting mass basis Lagrangian, normalisation constants Z_X and Feynman rules written in Mathematica format are stored on disk.

- Finally, the previously generated output can be used to export mass basis SMEFT interactions in various commonly used external formats such as Latex, WCxf and standard `FeynRules` supported interfaces – UFO, FeynArts and others. At this stage, users can choose the form of output parametrization, with Z_X normalisation constants also replaced by their corresponding explicit forms.

5. Deriving SMEFT Feynman rules with `SmeftFR` package

5.1. Model initialisation

In the first step, the relevant `FeynRules` model files must be generated. This is done by calling the function:

`SMEFTInitializeModel`[*Option1* $\rightarrow$ *Value1, Option2* $\rightarrow$ *Value2*, ...]

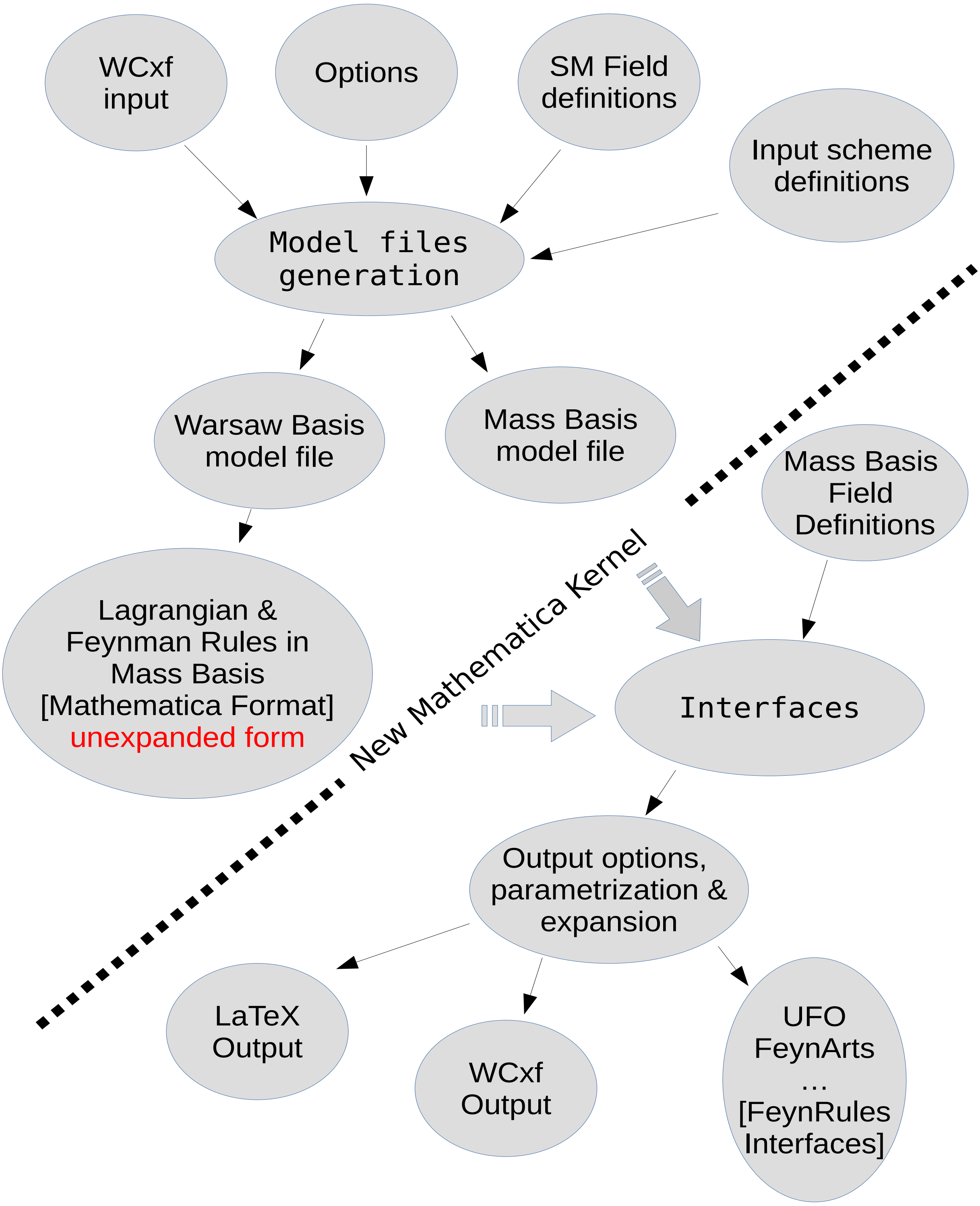


Figure 4: Structure of the SmeftFR v3 code.

Option	Allowed values	Description
Operators	list of operators	Subset of SMEFT operators included in calculations. Default: all $d = 5$ and $d = 6$ operators.
Gauge	**Unitary**, Rxi	Choice of gauge fixing conditions.
ExpansionOrder	0, **1** or 2	SMEFT interactions are expanded to $1/\Lambda^{2\,\mathrm{ExpansionOrder}}$ (default: $1/\Lambda^2$).
WCXFInitFile	**""**	Name of file with numerical values of Wilson coefficients in the WCxf format. If this option is not set, all WCs are initialised to 0.
RealParameters	**False**, True	Some codes like MadGraph 5 accept only real values of parameters. If this option is set to True, imaginary part of complex parameters are truncated in `FeynRules` model files.
InputScheme	**"GF"**, "AEM", ...	Selection of input parameters scheme, see discussion in Sec. 3.
CKMInput	"no", **"yes"**, "force"	Decides if corrections to CKM matrix are included (use "force" to add them even their relative size exceeds the threshold defined in variable `SMEFT$CKMTreshold` (default: 0.2).
MaxParticles	**6**	Only Feynman rules with less then MaxParticles external legs are calculated. Does not affect UFO and FeynArts output.
MajoranaNeutrino	**False**, True	Neutrinos are treated as Majorana spinors if $Q_{\nu\nu}$ is included in the operator list or this option is set to True, massless Weyl spinors otherwise.
Correct4Fermion	False, **True**	Corrects relative sign of some 4-fermion interactions, fixing results of `FeynRules` .
WBFirstLetter	**"c"**	Customisable first letter of Wilson coefficient names in Warsaw basis (default $c_G, \ldots$).
MBFirstLetter	**"C"**	Customisable first letter of Wilson coefficient names in mass basis (default $C_G, \ldots$).

Table 4: The allowed options of `SMEFTInitializeModel` routine. If an option is not specified, the default value (marked above in boldface) is assumed.

with the allowed options listed in Table 4.

The list and the naming of operators employed by `SmeftFR` v3 is arranged and explained in Appendix B. By default, all possible 59+1+4 SMEFT ($d = 5$ and $d = 6$) operator classes and no $d = 8$ operators are included in calculations, the latter can be added trivially by users if necessary.

To speed up the derivation of Feynman rules and to get more compact expressions, the user can restrict the list above to any preferred subset of operators (an example of initialisation with a sample operator subset is given in Sec. 6).

`SmeftFR` is fully integrated with the WCxf standard. Apart from numerically editing Wilson coefficients in `FeynRules` model files, reading them from the WCxf input is the only way of automatic initialisation of their numerical values. Such an input format is exchangeable between a larger set of SMEFT-related public packages [25] and helps in comparing their results.

An additional advantage of using WCxf input format comes in the flavour sector of the theory. Here, Wilson coefficients are in general tensors with flavour indices, in many cases symmetric under various permutations. WCxf input requires initialisation of only the minimal set of flavour dependent Wilson coefficients, those which could be derived by permutations are also automatically properly set.[4]

There is no commonly accepted standard for initialisation of numerical values of WCs of $d = 8$ operators, but as we only including scalar (no flavor indices) bosonic operators, adding them to WCxf-type input files is straightforward, we follow the convention for $d = 6$ bosonic operators just using the new names for $d = 8$ entries.

Further comments concern `MajoranaNeutrino` and `Correct4Fermion` options. They are used to modify the analytical expressions only for the Feynman rules, not at the level of the mass basis Lagrangian from which the rules are derived. This is because some `FeynRules` interfaces, like UFO, intentionally leave the relative sign of 4-fermion interactions uncorrected[5], as it is later changed by Monte-Carlo generators like MadGraph5. Correcting the sign before generating UFO output would therefore lead to wrong final result. Similarly, treatment of neutrinos as Majorana fields could not be compatible with hard coded quantum number definitions in various packages. On the other hand, in the manual or symbolic computations it is convenient to have from the start the correct form of Feynman rules, as done by `SmeftFR` when both options are set to their default values.

Currently, the predefined input scheme for initialisation of CKM matrix elements is based on the approach of ref. [15]. It can lead to numerically very large corrections to CKM matrix from some of the flavor off-diagonal 4-quark dimension-6 operators. Such large corrections usually mean that the assumed values of 4-quark WCs violate experimental bounds on flavor transitions and should be modified. In such case, by default `SmeftFR` v3 displays a relevant warning and does not include corrections to CKM matrix at all, expecting WC values to be

[4] We would like to thank D. Straub for supplying us with a code for symmetrization of flavour-dependent Wilson coefficients.

[5] B. Fuks, private communication.

modified. Such behaviour can be overwritten (so that even huge corrections are included, but the warning is still displayed) setting option `ForceCKMInput` $\to$ `True`. The maximum allowed size of corrections to CKM any of CKM elements is defined by variable `SMEFT$CKMTreshold` in the file `code/smeft_variables.m` and by default set to `SMEFT$CKMTreshold=0.2`. Users can modify this number to their preferred sensitivity level.

After execution, `SMEFTInitializeModel` creates in the `output` sub-directory two model files:

- `smeft_par_WB.fr`: SMEFT parameter file with Wilson coefficients in gauge basis (defined as "Internal", with no numerical values assigned).
- `smeft_par_MB.fr`: SMEFT parameter file with Wilson coefficients in mass basis (defined as "External", numerical values of WCs imported from the input file in WCxf format).

Note that field definitions are not generated dynamically and stored as fixed files named `smeft_fields_WB.fr` and `smeft_fields_MB.fr` in `definitions` sub-directory.

Parameter files generated by `SMEFTInitializeModel` contain also definitions of SM parameters, copied from several template files in `definitions` sub-directory and, most importantly, from the header of the `code/smeft_input_scheme.m` file, where the user-defined input parameters should be listed. Only the latter, values of user-defined parameters, are copied unchanged to model files, numerical values of other parameters can be updated to include corrections from higher order operators (thus hand-made modifications in files in `definitions` sub-directory are not advised and will be overwritten by the code).

As mentioned above, in all analytical calculations performed by `SmeftFR` , terms suppressed by terms of the order higher than $\mathcal{O}(1/\Lambda^{2\,\mathrm{ExpansionOrder}})$ are always neglected. Therefore, the resulting Feynman rules can be consistently used to calculate physical observables, symbolically or numerically by Monte-Carlo generators, up to the maximum quadratic order in dimension-6 operators and linear order in dimension-8 operators. This information is encoded in `FeynRules` SMEFT model files by assigning the "interaction order" parameter to Wilson coefficients: `NP=1` for $d = 6$ WCs and `NP=2` for $d = 8$ operators. `ExpansionOrder` parameter is passed also to model files `smeft_par_WB.fr` and `smeft_par_MB.fr` as:

```
M$InteractionOrderLimit = {
    {QCD,99},
    {NP,ExpansionOrder},
    {QED,99}
  }
```

5.2. Calculation of mass basis Lagrangian and Feynman rules

By loading the `FeynRules` model files, the derivation of SMEFT Lagrangian in mass basis is performed by calling the following sequence of routines:

`SMEFTLoadModel[ ]`	Loads `output/smeft_par_WB.par` model file and imports SMEFT Lagrangian in gauge basis for chosen subset of operators.
`SMEFTFindMassBasis[ ]`	Finds field bilinears and analytical transformations diagonalizing mass matrices. Calculates the expressions for Z_X normalisation constants.
`SMEFTFeynmanRules[ ]`	Evaluates analytically SMEFT Lagrangian and Feynman rules in the mass basis to a required order in $\mathcal{O}(1/\Lambda)$, *without substituting explicit expressions* for Z_X constants (see example in Fig. 1).
`SMEFTOutput[` *Options* `]`	By default stores SMEFT model file with parameters in mass basis as `output/smeft_par_MB.m` and mass basis Lagrangian and vertices in `output/smeft_feynman_rules.m`. To generate output in different locations, use options *ModelFile* $\to$ *filename1* and *TargetFile* $\to$ *filename2*.

The calculation time may vary considerably depending on the choice of operator (sub-)set and gauge fixing conditions chosen. For the full list of SMEFT $d = 5$ and $d = 6$ operators and in R_ξ-gauges, one can expect CPU time necessary to evaluate all Feynman rules for up to about an hour on a typical personal computer, depending on its speed capabilities. Adding $d = 8$ operators can obviously increase the CPU time, therefore it is advisable to choose only the operators relevant to a given analysis.

One should note that when neutrinos are treated as Majorana particles, (as necessary in case of non-vanishing Wilson coefficient of $d = 5$ Weinberg operator), their interactions involve lepton number non-conservation. Baryon and lepton (BL) number is also not conserved when explicitly BL-violating 4-fermion operators are included in Lagrangian. When `FeynRules` is dealing with such cases, it produces warnings of the form:

QN::NonConserv: Warning: non quantum number conserving vertex encountered!
Quantum number LeptonNumber not conserved in vertex ...

Obviously, such warnings in this specific case should be ignored.

Evaluation of Feynman rules for vertices involving more than two fermions is not fully implemented yet in `FeynRules`, and some warnings are displayed. To our experience, in most cases 4-fermion vertices are calculated correctly in spite of such warnings, apart from the issue of relative sign of four fermion diagrams mentioned earlier. Some cases are still problematic, e.g. the correct automatic derivation of quartic interactions with four Majorana neutrinos. For these special cases, `SmeftFR` overwrites the `FeynRules` result with manually calculated formulae encoded in Mathematica format.

Another remark concerns the hermicity property of the SMEFT Lagrangian. For some types of interactions, e.g. four-fermion vertices involving two-quarks and two-leptons, the function `CheckHermicity` provided by `FeynRules` reports non-Hermitian terms in the Lagrangian. However, such terms are actually Hermitian if permutation symmetries of indices of relevant Wilson coefficients are taken into account. Such symmetries are automatically imposed if numerical

`LeptonGaugeVertices`	`QuarkGaugeVertices`
`LeptonHiggsGaugeVertices`	`QuarkHiggsGaugeVertices`
`QuarkGluonVertices`	
`GaugeSelfVertices`	`GaugeHiggsVertices`
`GluonSelfVertices`	`GluonHiggsVertices`
`GhostVertices`	
`FourLeptonVertices`	`FourQuarkVertices`
`TwoQuarkTwoLeptonVertices`	
`DeltaLTwoVertices`	`BLViolatingVertices`

Table 5: Names of variables defined in the file `output/smeft_feynman_rules.m` containing expressions for Feynman rules. Parts of mass basis Lagrangian are stored in equivalent set of variables, with "`Vertices`" replaced by "`Lagrangian`" in part of their names (i.e. `LeptonGaugeVertices` → `LeptonGaugeLagrangian`, *etc.*).

values of Wilson coefficients are initialised with the use of `SMEFTInitializeMB` or `SMEFTToWCXF` routines (see Sec. 5.3 and 5.3.1).

Results of the calculations are by default collected in file `output/smeft_feynman_rules.m`. The Feynman rules and parts of the mass basis Lagrangian for various classes of interactions are stored in the variables with self-explanatory names listed in Table 5.

File `output/smeft_feynman_rules.m` contains also expressions for the normalisation factors Z_X relating Higgs and gauge fields and couplings in the Warsaw and mass basis, in "default" and "user" parametrizations (see Table 1 for corresponding names of code variables). In addition, formulae for tree level corrections to SM mass parameters and Yukawa couplings are stored in variables `SMEFT$vev`, `SMEFT$MH`, `SMEFT$MW`, `SMEFT$MZ`, `SMEFT$YL[i,j]`, `SMEFT$YD[i,j]` and `SMEFT$YU[i,j]`, as well as the selected user-defined program options.

As mentioned before, in expressions for Lagrangian parts and vertices stored in variables of Table 5 the Z_X constants are left in an unexpanded form, as in Fig. 1. To produce formulae fully expanded in $1/\Lambda$ powers to a required order, one must call the routine `SMEFTExpandVertices`, e.g. for vertices in "default" parametrization up to $1/\Lambda^4$ terms one should use

```
SMEFTExpandVertices[Input -> "smeft", ExpOrder -> 2]
```

(another possible choice is `Input → "user"`). Then expanded version of vertices is copied to variables ending with "Exp" (`LeptonGaugeVerticesExp, QuarkGaugeVerticesExp` etc.) and can be displayed or used in further calculations using standard `FeynRules` format.

At this point the Feynman rules for the mass basis Lagrangian are already calculated, but the definitions for fields and parameters used to initialise the SMEFT model in `FeynRules` are still given in gauge basis. To avoid inconsistencies, before exporting calculated expressions to other formats supported by `FeynRules` and `SmeftFR` one should quit the current Mathematica kernel and start a new one reloading the mass basis Lagrangian together with the compatible model files with fields defined also in mass basis, as described next in Sec. 5.3. All further calculations should be performed within this new kernel (routine `SMEFTExpandVertices` can be also used with this new kernel in the same way as described above).

5.3. Output formats and interfaces

`SmeftFR` output in some of the portable formats must be generated from the SMEFT Lagrangian transformed to mass basis, with all numerical values of parameters initialised. As `FeynRules` does not allow for two different model files loaded within a single *Mathematica* session, one needs to quit the kernel used to run routines necessary to obtain Feynman rules and, as described in the previous Section, start a new *Mathematica* kernel. Within it, the user must reload `FeynRules` and `SmeftFR` packages and call the following routine:

`SMEFTInitializeMB[` *Options* `]`

Allowed options are given in Table 6. After a call to `SMEFTInitializeMB`, mass basis model files are read and the mass basis Lagrangian is stored in a global variable named `SMEFT$MBLagrangian` for further use by interface routines.

5.3.1. WCxf input and output

Translation between `FeynRules` model files and WCxf format is done by the functions `SMEFTToWCXF` and `WCXFToSMEFT`. They can be used standalone and do not require loading `FeynRules` and calling first `SMEFTInitializeMB` routine to work properly.

Exporting numerical values of Wilson coefficients of operators in the WCxf format is done by the function:

`SMEFTToWCXF[ SMEFT_Parameter_File, WCXF_File,` *FirstLetter* → *SMEFT$MB]*

where the arguments `SMEFT_Parameter_File, WCXF_File` define the input model parameter file in the `FeynRules` format and the output file in the WCxf JSON format, respectively. Option `FirstLetter` denote the first letter of names of WCs in a parameter file and needs to be initialised only if it differs from variable `MBFirstLetter` in Table 4. The created JSON file can be used to transfer numerical values of Wilson coefficients to other codes supporting WCxf format. Note that in general, the `FeynRules` model files may contain different classes of parameters, according to the `Value` property defined to be a number (real or complex), a formula or even not defined at all. Only the Wilson coefficients with `Value` defined to be a number are transferred to the output file in WCxf format.

Conversely, files in WCxf format can be translated to `FeynRules` parameter files using two routines:

`ReadWCXFInput[ WCXF_File,` *Options* `]`
`WCXFToSMEFT[ SMEFT_Parameter_File,` *Options*`]`

where `ReadWCXFInput` reads values of WC from an input file in the WCxf format and `WCXFToSMEFT` creates parameter model file for `FeynRules` which contain all necessary entries, including, apart from WCs, also the definitions and numerical values of “default” and “user-defined” SMEFT input parameters. The allowed options for both routines defined in Table 7.

Option	Allowed values	Description
Expansion	“none”,**“smeft”**, “user”	Decides which parametrization is used to describe interaction vertices - with Z_X normalisation constants in an unexpanded form (“none”), using “default” SMEFT parameters (“smeft”) or user-defined set of parameters (“user”) (see Sec. 3.4 and examples in Figs. 1, 2, 3).
InteractionFile	*filename*	Name of the file with mass basis Lagrangian and vertices generated by `SMEFTOutput` routine. Default: `output/smeft_feynman_rules.m`
ModelFile	*filename*	Name of the model file containing SMEFT parameters in mass basis generated by `SMEFTOutput` routine. Default: `output/smeft_par_MB.fr`
Include4Fermion	False, **True**	4-fermion vertices are not fully supported by `FeynRules` - for extra safety calculations of them can be switched off by setting this option to False.
IncludeBL4Fermion	**False**, True	Baryon and lepton number violating 4-fermion vertices can be in principle evaluated by `FeynRules`, but including them may lead to compatibility problems with other codes - e.g. MadGraph 5 reports errors if such vertices are present in UFO file. Thus in `SmeftFR` evaluation of such vertices is by default switched off. Set this option to True to include them.

Table 6: Options of `SMEFTInitializeMB` routine, with default values marked in boldface.

5.3.2. Latex output

`SmeftFR` provides a dedicated Latex generator (not using the generic `FeynRules` Latex export routine). Its output has the following structure:

- For each interaction vertex, the diagram is drawn, using the `axodraw` style [91]. Expressions for Feynman rules are displayed next to corresponding diagrams.

Option	Allowed values	Description
Operators	default: all operators	List with subset of Wilson coefficients to be included in the SMEFT parameter file (`ReadWCXFInput` only)
RealParameters	False, True	Decides if only real values of Wilson coefficients given in WCxf file are included in SMEFT parameter file. The default value of this option is the same as set in the routine SMEFTInitializeModel, see Table 4.
OverwriteTarget	**False**, True	If set to True, target file is overwritten without warning

Table 7: Options of `ReadWCXFInput` and `WCXFToSMEFT` routines. Default values are marked in boldface. Options RealParameters and OverwriteTarget affect only `WCXFToSMEFT`.

- In analytical expressions, all terms multiplying a given Wilson coefficient are collected together and simplified.
- Long analytical expressions are automatically broken into many lines using `breakn` style (this does not always work perfectly but the printout is sufficiently readable).
- Latex output can be generated only for vertices expressed in terms of "default" SMEFT parameters, with Z_X constants expanded in terms of WCs or kept as symbols (corresponding to options "smeft" or "none" in Tables 6 and 8). This is because the simplification of Latex formulae is optimised for such particular parametrizations, vertices calculated in terms of completely general "user-defined" parameter set may not be well readable.
- Only terms up to maximal dimension 6 are included in Latex output. Again, as above, this is because including higher order terms leads in most cases to lengthy and not very readable expressions.

Latex output is generated by the function:

`SMEFTToLatex[` *Options* `]`

with the allowed options listed in Table 8. The function `SMEFTToLatex` assumes that the variables listed in Table 5 are initialised,thus it should be called after reloading the mass basis Lagrangian with the `SMEFTInitializeMB` routine, see Sec. 5.3.

Latex output is stored in `output/latex` sub-directory, split into smaller files, each containing one primary vertex. The main file is named `smeft_feynman_rules.tex`. The style files necessary to compile Latex output are supplied with the `SmeftFR` distribution.

Option name	Allowed values	Description
Expansion	**"none"**, "smeft"	Decides which parametrization is used to describe interaction vertices - with Z_X normalisation constants in an unexpanded form ("none") or using default SMEFT parameters ("smeft") (see discussion in Sec. 3.4 and examples in Figs. 1, 2,3).
FullDocument	False, **True**	By default a complete document is generated, with all headers necessary for compilation. If set to False, headers are stripped off and the output file can be, without modifications, included into other Latex documents.
ScreenOutput	**False**, True	For debugging purposes, if set to True the Latex output is printed also to the screen.

Table 8: Options of `SMEFTToLatex` routine, with default values marked in boldface.

Note that the correct compilation of documents using "axodraw.sty" style requires creating an intermediate Postscript file. Programs like *pdflatex* producing directly PDF output will not work properly. One should instead run in terminal in the correct directory e.g.:

```
latex smeft_feynman_rules.tex
dvips smeft_feynman_rules.dvi
ps2pdf smeft_feynman_rules.ps
```

or equivalent set of commands, depending on the Latex package used.

The `smeft_feynman_rules.tex` does not contain analytical expressions for five and six gluon vertices. Such formulae are very long (multiple pages, hard to even compile properly) and not useful for hand-made calculations. If such vertices are needed, they should be rather directly exported in some other formats, as described in the next subsection.

Other details not printed in the Latex output, such as, the form of field propagators, conventions for parameters and momenta flow in vertices (always incoming), manipulation of four-fermion vertices with Majorana fermions *etc*, are explained thoroughly in the Appendices A1–A3 of ref. [11].

5.3.3. FeynArts and analytical calculation tests

After calling the initialisation routine, `SMEFTInitializeMB`, one can generate output formats supported by native `FeynRules` interfaces, in particular one can export SMEFT interactions and parameters to files which could be imported by FeynArts (another especially important format, UFO, is discussed separately in the next section). For the descriptions of the available output formats and commands used to produce them, users should consult the

FeynRules manual. For instance, to generate FeynArts output for the full mass basis Lagrangian, one could call:

```
WriteFeynArtsOutput[ SMEFT$MBLagrangian, Output -> "output/FeynArts", ...]
```

It is important to note that FeynRules interfaces like FeynArts (or UFO described in Sec. 5.3.4), generate their output starting from the level of SMEFT mass basis Lagrangian. Thus, options of SMEFTInitializeModel function like MajoranaNeutrino and Correct4Fermion (see Table 4) have no effect on output generated by the interface routines. As explained in Sec. 5.1 they affect only the expressions for Feynman rules in FeynRules/Mathematica format (which are also used to generate Latex output file).

One should also note that FeynRules interfaces sometimes seem to be “non-commuting”. For example, calling FeynArts export routine first, may lead to errors in subsequent execution of UFO interfaces (like signalling problems with incorrect handling of vertices containing explicit $\sigma^{\mu\nu}$ Dirac matrices or issues with colour indices of SU(3) group structure constants), while the routines called in opposite order are both working properly. Therefore, it is safer to generate one type of FeynRules-supported output format at a time and reinitialise model in mass basis if more output types should be produced (WCxf and Latex generators does not suffer from such issues and can be safely used together with others).

Finally, we have tested that our Feynman rules communicate properly with FeynArts. An example of a non-trivial physics test we performed is the following: we used the programs' chain SmeftFR $\to$ FeynArts $\to$ FormCalc and calculated matrix elements for longitudinal vector boson scattering processes, $V_L V_L \to V_L V_L$ with $V = W^\pm, Z$ at tree level with the full set of $d = 6$ operators. According to the Goldstone-Boson-Equivalence Theorem (GBET) [92–95], at high energy this should be equal to the matrix elements for the Goldstone Boson scattering processes $GG \to GG$ where, $G = G^\pm, G^0$ which should only contain WCs associated to operators with powers of pure Higgs field φ and its derivatives. All other, and there are many, WCs cancel out non-trivially in all input ``user'' schemes employed by SmeftFR v3. Similarly, we have also checked the validity of GBET (at tree level) for $V_L V_L \to V_L V_L$ by including $d = 6$ and $d = 8$ operators involving the full set of pure Higgs boson operators and its derivatives.

It is perhaps instructive to provide one more test example for the dimension-8 operators: the positivity inequality constraints on WCs, see e.g. [96, 97]. According to analyticity of the amplitude, the Froissart bound, and the optical theorem, for any elastic 2-to-2 scattering amplitude $\mathcal{M}(ij \to ij)$ of SM particles i and j, the second derivative w.r.t the forward amplitude is positive semi-definite, i.e.,

$$\frac{d^2}{ds^2}\,\mathcal{M}(ij \to ij)(s, t = 0) \,\geq 0 \,, \tag{5.1}$$

where s, t are the Mandelstam variables.

By power counting, dimension-8 operators $Q^{(1,2,3)}_{\varphi^4 D^4}$, potentially affect the matrix elements between the Higgs (h) and the longitudinal components of the vector bosons (Z_L and/or $W_L^\pm$), by a factor s^2/Λ^4. This can be verified easily by using the FeynArt-output of SmeftFR and calculate the amplitudes with FormCalc. Then, application of (5.1) to the relevant matrix elements of the processes below, results in

$$hh \to hh \qquad \Longrightarrow \qquad C^{(1)}_{\varphi^4 D^4} + C^{(2)}_{\varphi^4 D^4} + C^{(3)}_{\varphi^4 D^4} \geq 0 \; , \tag{5.2}$$

$$Z_L h \to Z_L h \qquad \Longrightarrow \qquad C^{(2)}_{\varphi^4 D^4} \geq 0 \; , \tag{5.3}$$

$$W^+_L h \to W^+_L h \qquad \Longrightarrow \qquad C^{(1)}_{\varphi^4 D^4} + C^{(2)}_{\varphi^4 D^4} \geq 0 \; . \tag{5.4}$$

All other longitudinal vector boson elastic scattering amplitudes satisfy the above inequalities. For example, applying (5.1) to the amplitude $\mathcal{M}(W^+_L W^+_L \to W^+_L W^+_L)$ gives $C^{(1)}_{\varphi^4 D^4} + 2\, C^{(2)}_{\varphi^4 D^4} + C^{(3)}_{\varphi^4 D^4} \geq 0$, which is trivially satisfied by the inequalities (5.2)-(5.4). The above results are in agreement with ref. [96] and checked to be independent of the input-parameter-schemes used by `SmeftFR` .

Finally, several checks using Feynman Rules from `SmeftFR` with `FormCalc` or `FeynCalc` or by hand of various Ward-Identities have been performed, and we always found agreement.

5.3.4. UFO format and MadGraph 5 issues

Correct generation of UFO format requires more care. UFO format requires an extra parameter, “interaction order” (IO), to be assigned to all couplings, to help Monte-Carlo generators like MadGraph 5 decide the maximal order of diagrams included in amplitude calculations. It is customary to assign QED IO$= -1$ to Higgs boson VEV, v, as it is numerically a large number and multiplying by v can effectively cancel the suppression from smaller Yukawa or gauge couplings. In the SM, where all couplings are maximum dimension-4, such procedure never leads to total negative IO for any vertex. Unfortunately, in SMEFT some vertices are proportional to higher v powers and technically can have negative total “QED” interaction order, generating warnings when the UFO file is imported to MadGraph 5. However, all such vertices have simultaneously another type of IO assigned, “NP=0,1,2”, defining their EFT order (which is $1/\Lambda^{2\,\mathrm{NP}}$). The “NP” order is sufficient for MC generators to truncate the amplitude in a correct way, thus negative “QED” IO warnings can be ignored for such vertices. To avoid complications, `SmeftFR` v3 by default performs post-processing on UFO output files, removing “QED” IOs from all vertices proportional to WCs of higher dimension operators. Such post-processing can be switched off by setting the relevant option as described below.

Instead of `FeynRules`’s `WriteUFO` command, in `SmeftFR` v3 the UFO output format can be generated by calling the routine:

`SMEFTToUFO[ Lagrangian,` *Options* `]`

with options defined in Table 9. By default, argument `Lagrangian` should be set to variable named `SMEFT$MBLagrangian`, unless the user prefers to generate only interaction for some subsector of the theory, then it can be one of the variables defined in Table 5, with obvious name replacements like `LeptonGaugeVertices` $\to$ `LeptonGaugeLagrangian` etc.

One should note that some Monte-Carlo generators like MadGraph 5 support only real parameters, thus to generate UFO output working properly one should use option `RealParameters` $\to$ `True` when calling `SMEFTInitializeMB` routine. Also, MadGraph 5 has some hard coded names for QED and QCD coupling constants (`ee, aEWM1, aS`). For compatibility, `SmeftFR` v3 preserves those names, independently of how the "user-defined" input parameters are named (e.g., whatever is the name of the variable defining the strong coupling constant, it is always copied to `aS` used by MadGraph 5, and similarly for other "special" variables). If necessary for compatibility with other codes, more such "special" variable names could be added to the `SmeftFR` , editing the routine `UpdateSpecialParameters` in the file `smeft_parameters.m`.

Option name	Allowed values	Description
Output	**"output/UFO"**	default UFO output sub-directory, can be modified to other user-defined location.
CorrectIO	False, **True**	By default only "NP" interaction order parameter is left in vertices containing WCs of higher order operators. By setting this option to "False", preserves all IOs generated by native `FeynRules` UFO interface
AddDecays	**False**, True	UFO format can contain expressions for 2-body decays, switched off by default.

Table 9: Options of `SMEFTToUFO` routine, with default values marked in boldface.

If four-fermion vertices are included in SMEFT Lagrangian, the UFO generator produces warning messages of the form (similar warnings may appear also when using other `FeynRules` output routines):

Warning: Multi-Fermion operators are not yet fully supported!

Therefore, although in our experience it seems to work properly, the output for four-fermion interactions in UFO or other formats must be treated with care and limited trust — performing appropriate checks is left to users' responsibility.

Implementation in `FeynRules` of baryon and lepton number (BL) violating four-fermion interactions, with charge conjugation matrix appearing explicitly in vertices, is even more problematic. Thus, for safety in the current `SmeftFR` v3 such terms are by default not included in `SMEFT$MBLagrangian` variable, unless the option `IncludeBL4Fermion` in `SMEFTInitializeMB` routine is explicitly set to `True`. In such case, FeynArts output seems to work for such BL-violating vertices, but MadGraph 5 displays warnings that they are not yet supported and aborts process generation.

We have tested that `SmeftFR` works properly with MadGraph5. In particular, we ran without errors test simulations in MadGraph5 v3.4.1 using UFO model files produced by `SmeftFR` v3. Furthermore, we performed several types of numerical cross-checks against already existing codes:

- we compared cross-sections for various processes obtained with `SmeftFR` against the results obtained with `SMEFT@NLO` package up to terms of $\mathcal{O}(\Lambda^{-2})$ (note that `SMEFT@NLO`, Dim6Top and SMEFTsim have been formally validated up to this order [98], so it is sufficient to compare with only one of these codes),

- we compared matrix elements for various processes obtained with SmeftFR against the results obtained with SMEFTsim package up to terms of $\mathcal{O}(\Lambda^{-2})$, testing all implemented dimension 6 operators (apart from B- and L- violating ones),
- we compared matrix elements for various processes obtained with SmeftFR against the results obtained with the code based on [99] (available at https://feynrules.irmp.ucl.ac.be/wiki/AnomalousGaugeCoupling) up to terms of $\mathcal{O}(\Lambda^{-4})$, testing all operators considered in [99],

finding a very good agreement in each case.

For all comparisons which we performed we have used the (G_F, M_W, M_Z, M_H) input parameter scheme (option InputScheme $\to$ "GF" in SMEFTInitializeModel routine) with values of input parameters set to central values given in ref. [100] (unless explicitly stated otherwise below). In addition, CKM and PMNS matrices were approximated by unit matrices.

For cross-sections comparison, all particle widths, fermion masses and Yukawa couplings, except for the top quark, were assumed to be zero. Each cross section was calculated assuming that all but one Wilson coefficients were set to zero and the non-vanishing one (displayed in the left column of Table 10) had the value of $\left|\frac{C_i}{\Lambda^2}\right| = 10^{-6}$ GeV^{-2}, while its sign was always chosen to increase $\mathcal{O}(\Lambda^{-2})$ cross section w.r.t. SM. The results are summarised in the 2nd and 3rd column of Table 10. As one can see, differences between both codes at the $\mathcal{O}(\Lambda^{-2})$ level never exceed 1%.

The novel capability of SmeftFR v3 is the consistent inclusion of $O(1/\Lambda^4)$ terms in the interaction vertices. Therefore, SmeftFR v3 is able to *exactly* calculate dimension-6 squared terms in the amplitude. For completeness, we have checked the impact of such $\mathcal{O}(\Lambda^{-4})$ terms for the same processes. The corresponding cross sections can be found in the 4th column of the Table 10. The effect of higher order contributions is visible albeit small for the chosen small input values of WCs. However, in another example using SmeftFR with a large coefficient C_W displayed in Table 7 of ref. [101], the effect of dimension-6-squared terms on cross-section for W-boson scattering can be different by factors of thousand!

We have used similar procedure for matrix elements comparison. Once again each matrix element was calculated assuming that all but one Wilson coefficients were set to zero and the non-vanishing one had the value of $\frac{C_i^6}{\Lambda^2} = 10^{-6}$ GeV^{-2} for dimension-6 and $\frac{C_i^8}{\Lambda^2} = 10^{-11}$ GeV^{-4} for dimension-8 coefficients. We obtained almost identical results from SMEFTsim or AnomalousGaugeCoupling and SmeftFR for all of the studied processes, with the differences not exceeding 0.1%, usually being much smaller. As the number of compared processes is large, we do not include here the detailed comparison tables, they can be found on the SmeftFR homepage https://www.fuw.edu.pl/smeft.

5.4. Potential problems and optional further SmeftFR *extensions*

As already mentioned before, SMEFT itself is a very complicated model even at the level of Lagrangian construction. A transition amplitude calculations within SMEFT can easily

	SMEFT@NLO $\mathcal{O}(\Lambda^{-2})$	SmeftFR $\mathcal{O}(\Lambda^{-2})$	SmeftFR $\mathcal{O}(\Lambda^{-4})$
	$\mu^+\mu^- \to t\bar{t}$		
SM	0.16606 ± 0.00026	0.16608 ± 0.00024	-
C_{uW}^{33}	0.41862 ± 0.00048	0.41816 ± 0.00047	-
$C_{\varphi u}^{33}$	0.16725 ± 0.00027	0.16730 ± 0.00025	-
C_{lu}^{2233}	6.488 ± 0.016	6.491 ± 0.014	-
$C_{\varphi WB}$	0.21923 ± 0.00032	0.21940 ± 0.00030	0.22419 ± 0.00030
$C_{\varphi D}$	0.18759 ± 0.00030	0.18759 ± 0.00027	0.18829 ± 0.00027
	$\gamma\gamma \to t\bar{t}$		
SM	0.0037498 ± 0.0000050	0.0037498 ± 0.0000050	-
C_{uW}^{33}	0.008229 ± 0.000012	0.008235 ± 0.000012	-
$C_{\varphi WB}$	0.0053056 ± 0.0000086	0.0053056 ± 0.0000086	0.0055809 ± 0.0000090
$C_{\varphi D}$	0.0045856 ± 0.0000061	0.0045895 ± 0.0000064	0.0045882 ± 0.0000069
	$c\bar{c} \to t\bar{t}$		
SM	0.9553 ± 0.0017	0.9511 ± 0.0023	-
C_{uG}^{33}	1.1867 ± 0.0023	1.1854 ± 0.0021	-
C_{uW}^{33}	0.9641 ± 0.0018	0.9599 ± 0.0024	-
$C_{\varphi u}^{33}$	0.9555 ± 0.0017	0.9513 ± 0.0023	-
$C_{\varphi q3}^{33}$	0.9558 ± 0.0017	0.9515 ± 0.0023	-
C_{qu1}^{2233}	1.0111 ± 0.0018	1.0059 ± 0.0015	-
$C_{\varphi WB}$	0.9568 ± 0.0018	0.9520 ± 0.0018	0.9522 ± 0.0018
$C_{\varphi D}$	0.9558 ± 0.0017	0.9511 ± 0.0018	0.9511 ± 0.0018
	$pp \to t\bar{t}$		
SM	510.35 ± 0.72	510.46 ± 0.68	-
C_{uG}^{33}	664.33 ± 1.16	666.34 ± 0.90	671.08 ± 0.97
C_{uW}^{33}	510.63 ± 0.70	510.70 ± 0.80	-
$C_{\varphi u}^{33}$	510.37 ± 0.72	510.47 ± 0.68	-
$C_{\varphi q3}^{33}$	510.39 ± 0.72	510.65 ± 0.80	-
$\sum_{i=1,2} C_{qu1}^{ii33}$	516.31 ± 0.58	516.14 ± 0.64	-
$C_{\varphi WB}$	510.49 ± 0.68	510.52 ± 0.71	508.94 ± 0.79
$C_{\varphi D}$	510.38 ± 0.72	510.47 ± 0.68	508.89 ± 0.79

Table 10: Cross-sections (in pb) obtained using MadGraph5 with UFO models provided by SMEFTatNLO at the $\mathcal{O}(\Lambda^{-2})$ order of the EFT expansion and SmeftFR at the $\mathcal{O}(\Lambda^{-2})$ and $\mathcal{O}(\Lambda^{-4})$ orders of the EFT expansion for a chosen set of processes and SMEFT operators. An empty cell indicates that no $\mathcal{O}(\Lambda^{-4})$ terms appear in the amplitude.

increase the complexity of required analytical and numerical computations beyond the capability of humans or computers. Therefore, to remain within reasonable limits of time and effort required for a given analysis, it is strongly advised to generate necessary SMEFT interactions only for a subset of operators relevant to a chosen problem, a task for which SmeftFR was specifically designed for.

We performed number of tests to estimate the CPU time required to run the code for various initial SmeftFR v3 setups. Deriving Feynman rules in Mathematica/FeynRules format up to dimension-6 terms and for complete dimension-6 SMEFT Lagrangian (i.e. including 60 independent operators in Warsaw basis with fully general flavour structure and all numerical values of parameters initialised) takes about an hour on typical PC computer (depending on its speed of course), more if interaction vertices need to be expressed in terms of user-defined input parameters. Exporting Feynman rules to UFO or other formats is more time consuming, can take few or more hours. Including also all dimension-6 squared terms and the full set of bosonic dimension-8 operators at once does not seem to be feasible at all, as the computations can exhaust even large computer memory and/or human patience. For such calculations, choosing the subset of SMEFT operators is unavoidable.

Further problems related to complexity of SMEFT interactions, especially at the full dimension-8 level, may arise when importing the `SmeftFR` output to other public codes. In particular, in some cases we encountered difficulties when generating SMEFT processes with MadGraph5 - the Feynman rules in UFO file generated by `SmeftFR` contained such a lengthy expressions for interaction vertices that MadGraph internal compiler was unable to process them in a correct way and reported errors. Again, such issues could be solved (apart from using different Fortran or C compiler!) by limiting the number of included operators and/or decreasing the required order of EFT expansion to dimension-6 only.

6. Sample programs

After setting the variable `$FeynRulesPath` to the correct value, in order to evaluate mass basis SMEFT Lagrangian and analytical form of Feynman rules for some sample set of dimension-6 and 8 operators one can use the following sequence of commands:

```
SMEFT$MajorVersion = "3";
SMEFT$MinorVersion = "01";
SMEFT$Path = FileNameJoin[{$FeynRulesPath, "Models", "SMEFT_" <>
                    SMEFT$MajorVersion <> "_" <> SMEFT$MinorVersion}];

Get[ FileNameJoin[{$FeynRulesPath,"FeynRules.m"}] ];
Get[ FileNameJoin[{ SMEFT$Path, "code", "smeft_package.m"}] ];

OpList6 = {"phi", "phiBox", "phiD", "phiW", "phiWB", "eB", "uW", "dphi", "ll"};
OpList8 = {"phi8", "phi4n1", "phi4n3"};
OpList = Join[OpList6, OpList8];
```

```
 SMEFTInitializeModel[  Operators -> OpList,
                        Gauge -> Rxi,
                        WCXFInitFile -> "wcxf_input_file_with_path.json"
                        ExpansionOrder -> 1,
                        InputScheme -> "GF",
                        CKMInput -> "yes",
                        RealParameters -> True,
                        MaxParticles -> 4,
                        MajoranaNeutrino -> True,
                        Correct4Fermion -> False ];

SMEFTLoadModel[ ];
SMEFTFindMassBasis[ ];
SMEFTFeynmanRules[ ];
SMEFTOutput[ ];
```

or alternatively rerun the supplied programs: the notebook `SmeftFR-init.nb` or the text script `smeft_fr_init.m`.

After running the sequence of commands listed above, interaction vertices in different parametrizations become available and can be displayed on screen or used in further calculations. For example, the Higgs-photon-photon vertex for the fields in mass basis can be extracted in different schemes by using the commands:

```
Print["Higgs-photon-photon vertex in "none" scheme:  ",
SelectVertices[GaugeHiggsVertices, SelectParticles -> H, A, A]];

SMEFTExpandVertices[Input -> "smeft", ExpOrder -> 2];
Print["Higgs-photon-photon vertex in "smeft" scheme:  ",
SelectVertices[GaugeHiggsVerticesExp, SelectParticles -> H, A, A]];

SMEFTExpandVertices[Input -> "user", ExpOrder -> 2];
Print["Higgs-photon-photon vertex in "user" scheme:  ",
SelectVertices[GaugeHiggsVerticesExp, SelectParticles -> H, A, A]];
```

As described before, Latex, WCxf, UFO and FeynArts formats can be exported after rerunning first `SmeftFR-init.nb` or equivalent set of commands generating file `smeft_feynman_rules.m` containing the expressions for the mass basis Lagrangian. Then, the user needs to start a new *Mathematica* kernel and rerun the notebook file `SmeftFR-interfaces.nb` or the script `smeft_fr_interfaces.m`. Alternatively, one can manually type the commands, if necessary changing some of their options as described in previous Sections:

```
Get[ FileNameJoin[{$FeynRulesPath,"FeynRules.m"}] ];
Get[ FileNameJoin[{SMEFT$Path, "code", "smeft_package.m"}] ];

SMEFTInitializeMB[ Expansion->"user", Include4Fermion->True, ];

SMEFTToWCXF[ SMEFT$Path<>"output/smeft_par_MB.fr",
             SMEFT$Path<>"output/smeft_wcxf_MB.json" ];
```

```
SMEFToLatex[ Expansion -> "smeft" ];
SMEFTToUFO[ SMEFT$MBLagrangian, CorrectIO -> True, Output -> ... ];
WriteFeynArtsOutput[ SMEFT$MBLagrangian, Output -> ... ];
```

A step-by-step example on how to use `SmeftFR` v3 in practice is given in ref. [101].

7. Future Implementations

There are various important implementations that have been left out from the current version, `SmeftFR` v3, with the most pressing being the inclusion of fermionic dimension-8 operators. For instance, the latter have been proven recently [102] to provide dominant effects in vector-boson production. Unfortunately, including all such operators in full generality is difficult - they are numerous and implementing them correctly requires, comparing to pure bosonic case, taking into account their tensor structures in the flavour space, transformation properties under flavour rotations (necessary in transition to mass eigenstates basis), symmetry properties under flavour index permutations, etc. Apart from technical problems, computations involving large number of fermionic dimension-8 operators can exceed reasonable CPU running time and computer memory limits.

Nevertheless, as we have already mentioned, selected dimension-8 fermionic operators can be added to `SmeftFR` v3. However, it requires intervention in many parts of the code. For test purposes, we were able to successfully add a sample of dimension-8 fermionic operators to `SmeftFR` v3, and have documented the complete list of required code changes. At present, such prescription is rather complicated and requires knowledge of the internal code structure more detailed than can be expected from most users, so we decided not to include it in the current version of the manual. The file with the detailed instructions on how to do that is available on the web page of `SmeftFR` , `www.fuw.edu.pl/smeft`. If it is not sufficient, users interested in adding dimension-8 fermionic operators to `SmeftFR` can contact the authors for further help. We plan to include a simplified procedure of adding higher order fermionic operators in the next version of `SmeftFR` .

8. Summary

In recent years, SMEFT has become the standard framework for a concrete, robust, organised, and fairly model independent way of capturing physics beyond the SM. Huge efforts among the high energy community physicists, both theoretical and experimental, have been devoted to understand how to precisely map experimental observable and fit them onto the Wilson coefficients of the SMEFT Lagrangian in eq. 2.1. Even deriving the Feynman rules - a straightforward and most of the time effortless procedure in renormalizable theories - is not trivial in SMEFT: The abundance of operators and associated parameters, especially when climbing up in EFT-dimensionality, makes the computer aid necessary, if not indispensable.

In this paper, we present a new version of a code, the `SmeftFR` v3, previous versions of which had been tested in many work studies. `SmeftFR` v3 is able to express the SMEFT interaction vertices in terms of chosen, predefined or user-defined, set of observable input parameters, avoiding the need for reparametrizations required in calculations when expressed in terms of the SM gauge, Yukawa and Higgs coupling constants. One of `SmeftFR` v3 main advantages is that, it can calculate SMEFT interactions *à la carte* for user-defined subset of dimension-5, 6 and 8 operators, selected to be relevant to scattering matrix elements for observable (or observables) under scrutiny. It generates dynamically the corresponding `FeynRules` model files with the minimal required content, in effect producing more compact analytical formulae and significantly speeding up the numerical computations. The SMEFT Feynman rules can be calculated by `SmeftFR` v3 in unitary and R_ξ-gauges, following the procedure described in ref. [11]. A number of additional `SmeftFR` v3's options is described in details in this paper.

The output of the package can be printed in Latex or exported in various formats supported by `FeynRules`, such as UFO, FeynArts, *etc.* Input parameters for Wilson coefficients used in `SmeftFR` v3 can communicate with WCxf format for further numerical handling.

We have also performed a number of analytical and numerical consistency checks that came out from `SmeftFR` v3 calculations. Analytically, for example, we checked that the produced Feynman rules lead to correct non-trivial cancellations in Vector Boson Scattering helicity amplitudes in our predefined input-parameter schemes, certain Ward identities and positivity of combinations of dimension-8 Wilson coefficients. Numerically, we found very good agreement with other codes, such as `SMEFTsim` and `SMEFT@NLO`, commonly used for Monte-Carlo simulations in SMEFT. Compared to those codes, `SmeftFR` v3 offers in addition several important improvements: the precision of including consistently terms up to $O(1/\Lambda^4)$ (that is all (dimension-6)^2 terms and the full set of terms linear in WCs of bosonic dimension-8 operators), the physical input-parameter-schemes not only for the gauge and Higgs sector but also for the flavour sector by including SMEFT corrections to the CKM matrix, the inclusion of the SMEFT neutrino sector, and inclusion of the Baryon and Lepton number violating $d = 6$ interaction vertices.

The current version of `SmeftFR` v3 code and its manual can be downloaded from

`www.fuw.edu.pl/smeft`

We believe that `SmeftFR` v3 is an important tool, facilitating the computations within SMEFT from the theoretical Lagrangian level all the way down to amplitude calculations required by the beyond the SM physics experimental analyses.

Acknowledgements

The work of MR was supported in part by Polish National Science Centre under research grant DEC-2019/35/B/ST2/0200817. The research of JR has received funding from the Norwegian Financial Mechanism for years 2014-2021, under the grant no 2019/34/H/ST2/00707. The research work of LT was supported by the Hellenic Foundation for Research and Innovation (HFRI) under the HFRI PhD Fellowship grand (fellowship number: 1588). LT would like to thank the University of Warsaw for hospitality and financial support during his stay there. JR would like to thank the University of Ioannina and CERN for hospitality during his stays there. AD would like to thank CERN for hospitality. We would like to thank Dimitrios Beis for checking the SMEFT contributions to CKM and the numerical output of `SmeftFR` with an independent code.

Appendix A. Input schemes for the electroweak sector

The electroweak sector parameters, $\bar{g}$, $\bar{g}'$, v and λ, after expansion in $1/\Lambda$-powers can be written in the following form:

$$\begin{aligned}
\bar{g} &= \bar{g}_{SM} + \frac{1}{\Lambda^2}\bar{g}_{D6} + \frac{1}{\Lambda^4}\bar{g}_{D8} , \\
\bar{g}' &= \bar{g}'_{SM} + \frac{1}{\Lambda^2}\bar{g}'_{D6} + \frac{1}{\Lambda^4}\bar{g}'_{D8} , \\
v &= v_{SM} + \frac{1}{\Lambda^2}v_{D6} + \frac{1}{\Lambda^4}v_{D8} , \\
\lambda &= \lambda_{SM} + \frac{1}{\Lambda^2}\lambda_{D6} + \frac{1}{\Lambda^4}\lambda_{D8} .
\end{aligned} \tag{A.1}$$

where the exact form of "SM", "D6" and "D8" terms depends on the chosen input scheme. Below, we present relevant expressions for the two most commonly used SMEFT input schemes, both included as predefined routines in the `SmeftFR` v3 distribution.

A.1. *"GF" input scheme*

In this scheme Fermi constant G_F (evaluated from the muon lifetime measurement) and gauge and Higgs boson masses M_Z, M_W, M_H are used as the input parameters. To relate them to quantities defined in eq. (A.1), let us first define the following abbreviations

$$\begin{aligned}
\Delta M &= \sqrt{M_Z^2 - M_W^2} , \\
\mathcal{B}_6(C_{ll}, C_{\varphi l3}) &= -2(C_{ll}^{2112} - C_{\varphi l3}^{11} - C_{\varphi l3}^{22}) , \\
\mathcal{B}_8(C_{ll}, C_{\varphi l3}, C_{\varphi l1}) &= (C_{ll}^{2112})^2 + \frac{1}{4}(C_{le}^{2112})^2 - 2C_{ll}^{2112}C_{\varphi l3}^{11} - 2C_{ll}^{2112}C_{\varphi l3}^{22} \\
&+ (C_{\varphi l3}^{11})^2 + (C_{\varphi l3}^{22})^2 + 4C_{\varphi l3}^{11}C_{\varphi l3}^{22} \\
&+ C_{\varphi l1}^{21}C_{\varphi l3}^{12} - C_{\varphi l1}^{12}C_{\varphi l3}^{21} + C_{\varphi l1}^{12}C_{\varphi l1}^{21} - C_{\varphi l3}^{12}C_{\varphi l3}^{21} .
\end{aligned} \tag{A.2}$$

Then one can express quantities in eq. (A.1), as

$$\begin{aligned}
v_{SM} &= \frac{1}{2^{1/4}\sqrt{G_F}} , \\
v_{D6} &= \frac{v_{SM}}{4\sqrt{2}G_F}\mathcal{B}_6 , \\
v_{D8} &= \frac{v_{SM}}{64G_F^2}(\mathcal{B}_6^2 + 8\mathcal{B}_8) ,
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
\bar{g}_{SM} &= 2^{5/4}\sqrt{G_F}M_W , \\
\bar{g}_{D6} &= -\frac{\bar{g}_{SM}}{4\sqrt{2}G_F}\mathcal{B}_6 , \\
\bar{g}_{D8} &= \frac{\bar{g}_{SM}}{64G_F^2}(\mathcal{B}_6^2 - 8\mathcal{B}_8) ,
\end{aligned} \tag{A.4}$$

$$\bar{g}'_{SM} = 2^{5/4}\sqrt{G_F}\Delta M^2 ,$$

$$
\begin{aligned}
\bar{g}'_{D6} &= \frac{\bar{g}'_{SM}}{4\sqrt{2}G_F\Delta M}\left(-M_Z^2C_{\varphi D} - 4M_W\Delta M C_{\varphi WB} - \Delta M^2\mathcal{B}_6\right) , \\
\bar{g}'_{D8} &= \frac{\bar{g}'_{SM}}{16G_F^2\Delta M^2}\Big[-2M_Z^2(2C_{\varphi^6D^2} + \mathcal{B}_6C_{\varphi D}) + \Delta M^2(\mathcal{B}_6^2 - 8\mathcal{B}_8 - 16C_{\varphi WB}^2) \\
&- 8M_W\Big(2M_WC^{(3)}_{W^2\varphi^4} + 2\Delta M C^{(1)}_{WB\varphi^4} + \Delta M(\mathcal{B}_6 + 4C_{\varphi B} + 4C_{\varphi W})C_{\varphi WB}\Big)\Big] , \qquad \text{(A.5)}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{SM} &= \sqrt{2}G_FM_H^2 , \\
\lambda_{D6} &= \frac{\lambda_{SM}}{4G_F}\left[\frac{6}{G_FM_H^2}C_\varphi - \sqrt{2}\Big(\mathcal{B}_6 + 4C_{\varphi\Box} - C_{\varphi D}\Big)\right] , \\
\lambda_{D8} &= \frac{\lambda_{SM}}{16G_F^2}\left[\Big(\mathcal{B}_6^2 - 4\mathcal{B}_8 - 8C_{\varphi^6\Box} + 2C_{\varphi^2D^2}\Big) + \frac{6\sqrt{2}}{G_FM_H^2}\Big(\mathcal{B}_6C_\varphi + 2C_{\varphi 8}\Big)\right] . \qquad \text{(A.6)}
\end{aligned}
$$

A.2. "AEM" input scheme

In this scheme input parameters for the electroweak sector are chosen to be the electromagnetic coupling α_{em}, and the gauge and Higgs boson masses M_Z, M_W, M_H. Using again the abbreviation $\Delta M = \sqrt{M_Z^2 - M_W^2}$, for the quantities defined in eq. (A.1), one has:

$$
\begin{aligned}
v_{SM} &= \frac{M_W\Delta M}{M_Z\sqrt{\pi\alpha_{em}}} , \\
v_{D6} &= -\frac{\bar{g}_{SM}M_W^3}{4\pi\alpha_{em}M_Z^2}\left(M_WC_{\varphi D} + 4\Delta M C_{\varphi WB}\right) , \\
v_{D8} &= \frac{v_{SM}M_W^5}{32\pi^2\alpha_{em}^2M_Z^4}\Big[3M_W^3C_{\varphi D}^2 - 4M_W\Delta M^2C_{\varphi^6D^2} - 8(M_Z^2 - 5M_W^2)\Delta M C_{\varphi D}C_{\varphi WB} \\
&+ 16\Delta M^2\Big(4M_WC_{\varphi WB}^2 - \Delta M C^{(1)}_{WB\varphi^4} + \frac{M_Z^2 - 2M_W^2}{M_W}C^{(3)}_{WB\varphi^4}\Big) \\
&- 32\Delta M^3(C_{\varphi B} + C_{\varphi W})C_{\varphi WB}\Big] , \qquad \text{(A.7)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{g}_{SM} &= \frac{2M_Z\sqrt{\pi\alpha_{em}}}{\Delta M} , \\
\bar{g}_{D6} &= -v_{D6} , \\
\bar{g}_{D8} &= \frac{\bar{g}_{SM}M_W^5}{32\pi^2\alpha_{em}^2M_Z^4}\Big[-M_W^3C_{\varphi D}^2 + 4M_W\Delta M^2C_{\varphi^6D^2} + 8(M_Z^2 - 3M_W^2)\Delta M C_{\varphi D}C_{\varphi WB} \\
&- 16\Delta M^2\Big(2M_WC_{\varphi WB}^2 - \Delta M C^{(1)}_{WB\varphi^4} + \frac{M_Z^2 - 2M_W^2}{M_W}C^{(3)}_{WB\varphi^4}\Big) \\
&+ 32\Delta M^3(C_{\varphi B} + C_{\varphi W})C_{\varphi WB}\Big] , \qquad \text{(A.8)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{g}'_{SM} &= \frac{2M_Z\sqrt{\pi\alpha_{em}}}{M_W} , \\
\bar{g}'_{D6} &= -\frac{\bar{g}'_{SM}\Delta M^2M_W^2}{4\pi\alpha_{em}M_Z^2}C_{\varphi D} ,
\end{aligned}
$$

$$
\begin{aligned}
\bar{g}'_{D8} &= \frac{\bar{g}'_{SM} M_W^4 \Delta M^2}{32\pi^2 \alpha_{em}^2 M_Z^4}\Big[(M_W^2 + 3M_Z^2)C_{\varphi D}^2 - 16\Delta M^2 C_{\varphi WB}^2 + 16 M_W \Delta M C_{\varphi D} C_{\varphi WB} \\
&- 4\Delta M^2 \Big(C_{\varphi^6 D^2} + 4C^{(3)}_{W^2\varphi^4}\Big)\Big] , \qquad \text{(A.9)}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{SM} &= \frac{\pi \alpha_{em} M_H^2 M_Z^2}{\Delta M^2} , \\
\lambda_{D6} &= \frac{3\Delta M^2 M_W^2}{\pi \alpha_{em} M_Z^2} C_\varphi - 2M_H^2 C_{\varphi\Box} + \frac{M_H^2 M_Z^2}{2\Delta M^2} C_{\varphi D} + 2M_W \Delta M C_{\varphi WB} , \\
\lambda_{D8} &= \frac{M_W^2}{4\pi^2 \alpha_{em}^2 M_Z^4}\Big[12 M_W^2 \Delta M^4 2C_{\varphi 8} - 6M_W^3 \Delta M^2 C_\varphi (M_W C_{\varphi D} + 4\Delta M C_{\varphi WB}) \\
&+ \pi \alpha_{em} M_Z^2 M_H^2 \Big(-4\Delta M^2 C_{\varphi^6 \Box} + M_Z^2 C_{\varphi^2 D^2} + 8 M_W \Delta M (C_{\varphi B} + C_{\varphi W}) C_{\varphi WB} \\
&+ \frac{2M_W (M_Z^2 - 2M_W^2)}{\Delta M} C_{\varphi D} C_{\varphi WB} - 4M_W^2 C_{\varphi WB}^2 \\
&+ 4 M_W \Delta M C^{(1)}_{WB\varphi^4} - 4(M_Z^2 - 2M_W^2) C^{(3)}_{WB\varphi^4}\Big)\Big] . \qquad \text{(A.10)}
\end{aligned}
$$

Appendix B. Operators and their naming used in `SmeftFR`

All dimension-6 operators in Warsaw basis are given in Table B.1 (copied here for complementarity from ref. [8]). Naming of `SmeftFR` variables corresponding to WCs of these operators is straightforward: each variable name consists of subscripts identifying a given operator, with obvious transcriptions of "tilde" symbol and Greek letters to Latin alphabet. Operator names are represented by strings, to avoid accidental use of similarly named variables for other purposes. For example, one may include in `OpList6` (list of dimension-6 operators, see examples in Sec. 6):

$$
\begin{aligned}
Q_\varphi &\to \text{"phi"} \\
Q_{\varphi D} &\to \text{"phiD"} \\
Q_{\varphi\Box} &\to \text{"phiBox"} \\
Q_{\varphi\widetilde{W}} &\to \text{"phiWtilde"} \\
Q^{(3)}_{lq} &\to \text{"lq3"} \\
Q^{(8)}_{quqd} &\to \text{"quqd8"} \\
&\ldots.
\end{aligned}
$$

The full list of all dimension-6 operators contains the following entries:

```
OpList6 = { "G", "Gtilde", "W", "Wtilde", "phi", "phiBox", "phiD", "phiW", "phiB", "phiWB",
"phiWtilde", "phiBtilde", "phiWtildeB", "phiGtilde", "phiG", "ephi", "dphi", "uphi", "eW",
"eB", "uG", "uW", "uB", "dG", "dW", "dB", "phil1", "phil3", "phie", "phiq1", "phiq3", "phiu",
"phid", "phiud", "ll", "qq1", "qq3", "lq1", "lq3", "ee", "uu", "dd", "eu", "ed", "ud1", "ud8",
"le", "lu", "ld", "qe", "qu1", "qu8", "qd1", "qd8", "ledq", "quqd1", "quqd8", "lequ1", "lequ3",
"vv", "duq", "qqu", "qqq", "duu" }
```

Similarly, `SmeftFR` takes as input bosonic dimension-8 operators from Tables B.2, B.3, B.4, again rewritten here for completeness from ref. [9]. For example, one can use the following names in the list of dimension-8 operators:

$$
\begin{aligned}
Q^{(1)}_{\varphi^4 D^4} &\to \text{“}\texttt{phi4D4n1}\text{”}\\
Q_{\varphi^6 \Box} &\to \text{“}\texttt{phi6Box}\text{”}\\
Q^{(4)}_{G^2 B^2} &\to \text{“}\texttt{G2B2n4}\text{”}\\
Q^{(2)}_{W^2 B \varphi^2} &\to \text{“}\texttt{W2Bphi2n2}\text{”}\\
Q^{(1)}_{W^2 \varphi^2 D^2} &\to \text{“}\texttt{W2phi2D2n1}\text{”}\\
&\ldots.
\end{aligned}
$$

Table B.2 collects the pure Higgs operators, i.e. operators constructed only out of the Higgs doublet, φ, and covariant derivatives. There, we performed a change of basis in the operators of the $\varphi^6 D^2$ class so that they have immediate connection with the Warsaw basis. The original operators where defined in [9] as

$$
\begin{aligned}
Q^{(1)}_{\varphi^6} &= (\varphi^\dagger \varphi)^2 (D_\mu \varphi^\dagger D^\mu \varphi)\,,\\
Q^{(2)}_{\varphi^6} &= (\varphi^\dagger \varphi)(\varphi^\dagger \tau^I \varphi)(D_\mu \varphi^\dagger \tau^I D^\mu \varphi)\,,
\end{aligned}
$$

and here we use instead the set

$$
\begin{aligned}
Q_{\varphi^6 \Box} &= (\varphi^\dagger \varphi)^2 \Box (\varphi^\dagger \varphi)\,,\\
Q_{\varphi^6 D^2} &= (\varphi^\dagger \varphi)(\varphi^\dagger D_\mu \varphi)^* (\varphi^\dagger D^\mu \varphi)\,,
\end{aligned}
$$

which naturally extends the definition of the dimension 6 operators $Q_{\varphi\Box}$ and $Q_{\varphi D}$ from table B.1. This change of basis is consistent with the rest of the basis from ref. [9]. A proof of this result can be found in appendix F of ref. [103] for any order in the EFT expansion. Additionally, we added the number of covariant derivatives in the naming of the operators that belong in the third class, $\varphi^4 D^4$, to avoid confusion with the SM quartic Higgs operator, φ^4.

Table B.3 collects the operators that are constructed purely from gauge field strengths. Therefore, each operator there contains exactly four field strengths, and the operator classes are further divided as X^4, where only one of the field strengths of the B, W or G gauge fields appears in the operator, $X^3 X'$, where the G field strength appears thrice together with a B field strength in the operator, and finally $X^2 X'^2$, where the operators are consisted of two pairs of different field strengths. The notation in this table follows exactly ref. [9]. Finally, table B.4 collects the operators that are constructed from a combination of Higgs doublets, φ, and gauge field strengths.

The full list of names of bosonic dimension-8 operators in the basis of ref. [9] (with the modifications described above) which can be included in `SmeftFR` v3 calculations reads as:

```
OpList8 = { "phi8", "phi6Box", "phi6D2", "G2phi4n1", "G2phi4n2", "W2phi4n1", "W2phi4n2",
"W2phi4n3", "W2phi4n4", "WBphi4n1", "WBphi4n2", "B2phi4n1", "B2phi4n2", "G4n1", "G4n2",
"G4n3", "G4n4", "G4n5", "G4n6", "G4n7", "G4n8", "G4n9", "W4n1", "W4n2", "W4n3", "W4n4", "W4n5",
```

"W4n6", "B4n1", "B4n2", "B4n3", "G3Bn1", "G3Bn2", "G3Bn3", "G3Bn4", "G2W2n1", "G2W2n2", "G2W2n3", "G2W2n4", "G2W2n5", "G2W2n6", "G2W2n7", "G2B2n1", "G2B2n2", "G2B2n3", "G2B2n4", "G2B2n5", "G2B2n6", "G2B2n7", "W2B2n1", "W2B2n2", "W2B2n3", "W2B2n4", "W2B2n5", "W2B2n6", "W2B2n7", "phi4D4n1", "phi4D4n2", "phi4D4n3", "G3phi2n1", "G3phi2n2", "W3phi2n1", "W3phi2n2", "W2Bphi2n1", "W2Bphi2n2", "G2phi2D2n1", "G2phi2D2n2", "G2phi2D2n3", "W2phi2D2n1", "W2phi2D2n2", "W2phi2D2n3", "W2phi2D2n4", "W2phi2D2n5", "W2phi2D2n6", "WBphi2D2n1", "WBphi2D2n2", "WBphi2D2n3", "WBphi2D2n4", "WBphi2D2n5", "WBphi2D2n6", "B2phi2D2n1", "B2phi2D2n2", "B2phi2D2n3", "Wphi4D2n1", "Wphi4D2n2", "Wphi4D2n3", "Wphi4D2n4", "Bphi4D2n1", "Bphi4D2n2" }

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

Table B.1: The full set of dimension 6 operators in Warsaw basis [8]. The sub-tables in the two upper rows collect all operators except the four-fermion ones, which are collected separately in the sub-tables of the two bottom rows.

φ^8		$\varphi^6 D^2$		$\varphi^4 D^4$	
Q_{φ^8}	$(\varphi^\dagger\varphi)^4$	$Q_{\varphi^6\Box}$	$(\varphi^\dagger\varphi)^2\Box(\varphi^\dagger\varphi)$	$Q^{(1)}_{\varphi^4 D^4}$	$(D_\mu\varphi^\dagger D_\nu\varphi)(D^\nu\varphi^\dagger D^\mu\varphi)$
		$Q_{\varphi^6 D^2}$	$(\varphi^\dagger\varphi)(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)$	$Q^{(2)}_{\varphi^4 D^4}$	$(D_\mu\varphi^\dagger D_\nu\varphi)(D^\mu\varphi^\dagger D^\nu\varphi)$
				$Q^{(3)}_{\varphi^4 D^4}$	$(D_\mu\varphi^\dagger D^\mu\varphi)(D_\nu\varphi^\dagger D^\nu\varphi)$

Table B.2: Dimension 8 operators containing only the Higgs field. Table taken from ref. [9] except for the two operators in $\varphi^6 D^2$ class that have been modified as discussed in this Appendix.

$X^4,\ X^3X'$		$X^2X'^2$	
$Q^{(1)}_{G^4}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$	$Q^{(1)}_{G^2W^2}$	$(W^I_{\mu\nu}W^{I\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(2)}_{G^4}$	$(G^A_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(2)}_{G^2W^2}$	$(W^I_{\mu\nu}\widetilde{W}^{I\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(3)}_{G^4}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$	$Q^{(3)}_{G^2W^2}$	$(W^I_{\mu\nu}G^{A\mu\nu})(W^I_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(4)}_{G^4}$	$(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(4)}_{G^2W^2}$	$(W^I_{\mu\nu}\widetilde{G}^{A\mu\nu})(W^I_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(5)}_{G^4}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(5)}_{G^2W^2}$	$(W^I_{\mu\nu}\widetilde{W}^{I\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(6)}_{G^4}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(6)}_{G^2W^2}$	$(W^I_{\mu\nu}W^{I\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(7)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$	$Q^{(7)}_{G^2W^2}$	$(W^I_{\mu\nu}G^{A\mu\nu})(W^I_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(8)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$	$Q^{(1)}_{G^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(9)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$	$Q^{(2)}_{G^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(1)}_{W^4}$	$(W^I_{\mu\nu}W^{I\mu\nu})(W^J_{\rho\sigma}W^{J\rho\sigma})$	$Q^{(3)}_{G^2B^2}$	$(B_{\mu\nu}G^{A\mu\nu})(B_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(2)}_{W^4}$	$(W^I_{\mu\nu}\widetilde{W}^{I\mu\nu})(W^J_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(4)}_{G^2B^2}$	$(B_{\mu\nu}\widetilde{G}^{A\mu\nu})(B_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(3)}_{W^4}$	$(W^I_{\mu\nu}W^{J\mu\nu})(W^I_{\rho\sigma}W^{J\rho\sigma})$	$Q^{(5)}_{G^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(4)}_{W^4}$	$(W^I_{\mu\nu}\widetilde{W}^{J\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(6)}_{G^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(5)}_{W^4}$	$(W^I_{\mu\nu}W^{I\mu\nu})(W^J_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(7)}_{G^2B^2}$	$(B_{\mu\nu}G^{A\mu\nu})(B_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(6)}_{W^4}$	$(W^I_{\mu\nu}W^{J\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(1)}_{W^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(W^I_{\rho\sigma}W^{I\rho\sigma})$
$Q^{(1)}_{B^4}$	$(B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}B^{\rho\sigma})$	$Q^{(2)}_{W^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(2)}_{B^4}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(B_{\rho\sigma}\widetilde{B}^{\rho\sigma})$	$Q^{(3)}_{W^2B^2}$	$(B_{\mu\nu}W^{I\mu\nu})(B_{\rho\sigma}W^{I\rho\sigma})$
$Q^{(3)}_{B^4}$	$(B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}\widetilde{B}^{\rho\sigma})$	$Q^{(4)}_{W^2B^2}$	$(B_{\mu\nu}\widetilde{W}^{I\mu\nu})(B_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(1)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{C\rho\sigma})$	$Q^{(5)}_{W^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(W^I_{\rho\sigma}W^{I\rho\sigma})$
$Q^{(2)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{C\rho\sigma})$	$Q^{(6)}_{W^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(3)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}G^{C\rho\sigma})$	$Q^{(7)}_{W^2B^2}$	$(B_{\mu\nu}W^{I\mu\nu})(B_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(4)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{C\rho\sigma})$		

Table B.3: Dimension 8 operators containing only gauge field strengths. Table taken from ref. [9].

$X^3\varphi^2$		$X^2\varphi^4$	
$Q^{(1)}_{G^3\varphi^2}$	$f^{ABC}(\varphi^\dagger\varphi)G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$Q^{(1)}_{G^2\varphi^4}$	$(\varphi^\dagger\varphi)^2 G^A_{\mu\nu}G^{A\mu\nu}$
$Q^{(2)}_{G^3\varphi^2}$	$f^{ABC}(\varphi^\dagger\varphi)G^{A\nu}_\mu G^{B\rho}_\nu \widetilde{G}^{C\mu}_\rho$	$Q^{(2)}_{G^2\varphi^4}$	$(\varphi^\dagger\varphi)^2 \widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$
$Q^{(1)}_{W^3\varphi^2}$	$\epsilon^{IJK}(\varphi^\dagger\varphi)W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$	$Q^{(1)}_{W^2\varphi^4}$	$(\varphi^\dagger\varphi)^2 W^I_{\mu\nu}W^{I\mu\nu}$
$Q^{(2)}_{W^3\varphi^2}$	$\epsilon^{IJK}(\varphi^\dagger\varphi)W^{I\nu}_\mu W^{J\rho}_\nu \widetilde{W}^{K\mu}_\rho$	$Q^{(2)}_{W^2\varphi^4}$	$(\varphi^\dagger\varphi)^2 \widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$
$Q^{(1)}_{W^2B\varphi^2}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)B^{\ \nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$	$Q^{(3)}_{W^2\varphi^4}$	$(\varphi^\dagger\tau^I\varphi)(\varphi^\dagger\tau^J\varphi)W^I_{\mu\nu}W^{J\mu\nu}$
$Q^{(2)}_{W^2B\varphi^2}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)(\widetilde{B}^{\mu\nu}W^J_{\nu\rho}W^{K\rho}_\mu + B^{\mu\nu}W^J_{\nu\rho}\widetilde{W}^{K\rho}_\mu)$	$Q^{(4)}_{W^2\varphi^4}$	$(\varphi^\dagger\tau^I\varphi)(\varphi^\dagger\tau^J\varphi)\widetilde{W}^I_{\mu\nu}W^{J\mu\nu}$
		$Q^{(1)}_{WB\varphi^4}$	$(\varphi^\dagger\varphi)(\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu}$
		$Q^{(2)}_{WB\varphi^4}$	$(\varphi^\dagger\varphi)(\varphi^\dagger\tau^I\varphi)\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$
		$Q^{(1)}_{B^2\varphi^4}$	$(\varphi^\dagger\varphi)^2 B_{\mu\nu}B^{\mu\nu}$
		$Q^{(2)}_{B^2\varphi^4}$	$(\varphi^\dagger\varphi)^2 \widetilde{B}_{\mu\nu}B^{\mu\nu}$

$X^2\varphi^2D^2$		$X\varphi^4D^2$	
$Q^{(1)}_{G^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D^\nu\varphi)G^A_{\mu\rho}G^{A\rho}_\nu$	$Q^{(1)}_{W\varphi^4D^2}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)W^I_{\mu\nu}$
$Q^{(2)}_{G^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D_\mu\varphi)G^A_{\nu\rho}G^{A\nu\rho}$	$Q^{(2)}_{W\varphi^4D^2}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)\widetilde{W}^I_{\mu\nu}$
$Q^{(3)}_{G^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D_\mu\varphi)G^A_{\nu\rho}\widetilde{G}^{A\nu\rho}$	$Q^{(3)}_{W\varphi^4D^2}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)(D^\mu\varphi^\dagger\tau^J D^\nu\varphi)W^K_{\mu\nu}$
$Q^{(1)}_{W^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D^\nu\varphi)W^I_{\mu\rho}W^{I\rho}_\nu$	$Q^{(4)}_{W\varphi^4D^2}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)(D^\mu\varphi^\dagger\tau^J D^\nu\varphi)\widetilde{W}^K_{\mu\nu}$
$Q^{(2)}_{W^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D_\mu\varphi)W^I_{\nu\rho}W^{I\nu\rho}$	$Q^{(1)}_{B\varphi^4D^2}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger D^\nu\varphi)B_{\mu\nu}$
$Q^{(3)}_{W^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D_\mu\varphi)W^I_{\nu\rho}\widetilde{W}^{I\nu\rho}$	$Q^{(2)}_{B\varphi^4D^2}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger D^\nu\varphi)\widetilde{B}_{\mu\nu}$
$Q^{(4)}_{W^2\varphi^2D^2}$	$i\epsilon^{IJK}(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)W^J_{\mu\rho}W^{K\rho}_\nu$		
$Q^{(5)}_{W^2\varphi^2D^2}$	$\epsilon^{IJK}(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu - \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$		
$Q^{(6)}_{W^2\varphi^2D^2}$	$i\epsilon^{IJK}(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$		
$Q^{(1)}_{WB\varphi^2D^2}$	$(D^\mu\varphi^\dagger\tau^I D_\mu\varphi)B_{\nu\rho}W^{I\nu\rho}$		
$Q^{(2)}_{WB\varphi^2D^2}$	$(D^\mu\varphi^\dagger\tau^I D_\mu\varphi)B_{\nu\rho}\widetilde{W}^{I\nu\rho}$		
$Q^{(3)}_{WB\varphi^2D^2}$	$i(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}W^{I\rho}_\nu - B_{\nu\rho}W^{I\rho}_\mu)$		
$Q^{(4)}_{WB\varphi^2D^2}$	$(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}W^{I\rho}_\nu + B_{\nu\rho}W^{I\rho}_\mu)$		
$Q^{(5)}_{WB\varphi^2D^2}$	$i(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}\widetilde{W}_\nu^{I\ \rho} - B_{\nu\rho}\widetilde{W}_\mu^{I\ \rho})$		
$Q^{(6)}_{WB\varphi^2D^2}$	$(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}\widetilde{W}_\nu^{I\ \rho} + B_{\nu\rho}\widetilde{W}_\mu^{I\ \rho})$		
$Q^{(1)}_{B^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D^\nu\varphi)B_{\mu\rho}B_\nu^{\ \rho}$		
$Q^{(2)}_{B^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D_\mu\varphi)B_{\nu\rho}B^{\nu\rho}$		
$Q^{(3)}_{B^2\varphi^2D^2}$	$(D^\mu\varphi^\dagger D_\mu\varphi)B_{\nu\rho}\widetilde{B}^{\nu\rho}$		

Table B.4: Dimension 8 operators containing both gauge field strengths and the Higgs field. Table taken (and modified according to our notation) from ref. [9].

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