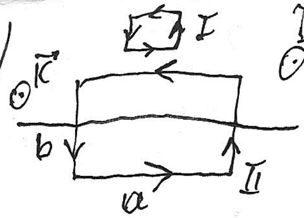
seria 6

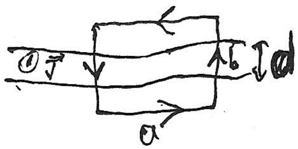
1) a/  $\overset{z}{\uparrow}\!\to y$ $\odot x$ $\odot \vec K$ prawo Amperea: $\iint \mu_0 \vec J\, d\vec S = \oint \vec B\, d\vec l$

$\vec B \perp \vec J$ — nie ma B_x

z rameczki I i symetrii nie ma B_z

$\mu_0 \iint \vec J\, d\vec s = \int_0^a \mu_0 \vec K \cdot \vec n\, dl = \mu_0 K \cdot a$; $\oint \vec B\, d\vec l = 2a \cdot B_y$

$B_y = \frac{\mu_0 K}{2} \to \vec B = \begin{cases} -\mu_0 K/2\, \hat e_y & y>0 \\ \mu_0 K/2\, \hat e_y & y<0 \end{cases}$

b)  analogicznie $\vec B = B_y \hat e_y$

$\mu_0 \iint \vec J\, d\vec S = \mu_0 J a \int_{-d}^{d} dz = 2aB_y \to B_y = \begin{cases} \frac{\mu_0 J d}{2} & d<b \\ \frac{\mu_0 J b}{2} & d>b \end{cases}$

$\vec B = \begin{cases} -\mu_0 J \frac{d}{2} & r>d \\ -\mu_0 J \frac{r}{2} & 0<r<d \end{cases}$, $\mu_0 J\, r/2$ $\; -d<r<0$, $\mu_0 J\, d/2$ $\; r<-d$

2) 

$\vec B \perp \vec J$ — nie ma B_z; z symetrii $B_r = 0$

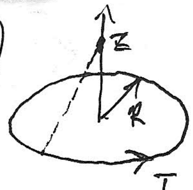
$\vec B = B \hat e_\varphi$; $\mu_0 \iint \vec J\, d\vec S = 2\pi \mu_0 \int_0^{(r,R)} j(p) \cdot p\, dp$

$\oint \vec B\, d\vec l = 2\pi r B \to \vec B = \frac{\mu_0}{r} \int_0^{(r,R)} j(p) \cdot p\, dp\; \hat e_\varphi$

a) $j(p) = J_0 \to B = \frac{\mu_0 J_0}{r} \int_0^{(r,R)} p\, dp = \begin{cases} \mu_0 J_0\, r/2 & r<R \\ \frac{\mu_0 J_0 R^2}{2r} & r>R \end{cases}$

b) $j(p) = kp \to B = \begin{cases} \frac{\mu_0 k r^2}{3} & r<R \\ \frac{\mu_0 k R^3}{3r} & r>R \end{cases}$

3)

3) 

$\vec{r} = (0,0,z)$, $\vec{r}\,' = (R\cos\varphi, R\sin\varphi, 0) \rightarrow \vec{r} - \vec{r}\,' = z\hat{e}_z - R\hat{e}_r$

$d\vec{I} = I\,d\vec{l} = I\frac{d\vec{l}}{d\varphi}d\varphi = IR\,d\varphi\cdot\hat{e}_\varphi$ $\qquad |\vec{r}-\vec{r}\,'| = \sqrt{R^2+z^2}$

$d\vec{I}\times(\vec{r}-\vec{r}\,') = (IRz\,\hat{e}_r + IR^2\hat{e}_z)\,d\varphi$

$\hat{e}_r = \cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y \longrightarrow \int_0^{2\pi}\cos\varphi\,d\varphi = \int_0^{2\pi}\sin\varphi\,d\varphi = 0 \Longrightarrow \int_0^{2\pi}\hat{e}_r\,d\varphi = 0$

$$\vec{B} = \frac{\mu_0}{4\pi}\int_0^{2\pi}\frac{IR^2\hat{e}_z}{\sqrt{R^2+z^2}^{\,3}}\,d\varphi = \frac{\mu_0 I R^2 e_z}{2\sqrt{R^2+z^2}^{\,3}}$$

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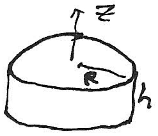
4) 

$\vec{M} = M\hat{e}_z \qquad \vec{J}_b = \vec{\nabla}\times\vec{M} = 0 \qquad \vec{K}_b(\text{podst.}) = \vec{M}\times\vec{n}_p = \pm M\hat{e}_z\times\hat{e}_z = 0$

$\vec{K}_b(\text{bok}) = \vec{M}\times\vec{n}_b = M\hat{e}_z\times\hat{e}_r = M\hat{e}_\varphi$

W dużej odległości zaniedbamy wysokość cylindra () i zrobimy przybliżenie multipolowe

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\oint\frac{d\vec{I}}{|\vec{r}-\vec{r}\,'|} = \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint(r')^n P_n(\cos\theta)\,d\vec{I}$$

$n=0 \rightarrow \oint d\vec{I} = I\oint d\vec{l} = 0$ (bo zamknięty obieg)

$n=1 \rightarrow \oint_{\partial S} r'\cos\theta\,d\vec{I} = I\oint_{\partial S}\vec{r}\,'\cdot d\vec{r}\,' = \left|\begin{array}{l} d\vec{I} = I\,d\vec{r}\,' \\ \vec{r}\,' = (R\cos\varphi, R\sin\varphi, 0) \\ d\vec{r}\,' = (R\sin\varphi, R\cos\varphi, 0)\end{array}\right| = I\iint_S d\vec{s}\times\vec{r}$

$\cancel{\vec{A}_1 = \frac{\mu_0 I}{4\pi r^2}\vec{S}\times\hat{e}_r = \frac{\mu_0}{4\pi r^2}\vec{m}\times\hat{e}_r,\ \text{gdzie}\ \vec{m} = I\vec{S}}$

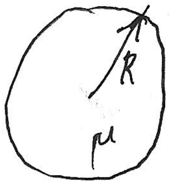
W naszym wypadku $\vec{m} = I\cdot\pi R^2\,\hat{e}_z \rightarrow \vec{A} = \frac{\mu_0}{4\pi r^3}\vec{m}\times\vec{r}$

$\cancel{\vec{A} = \frac{\mu_0}{4\pi r^2}I\pi R^2\,\hat{e}_z\times\hat{e}_r = \frac{\mu_0 I R^2}{4 r^2}}$ $\qquad \vec{A} = \frac{\mu_0 m}{4\pi r^2}\hat{e}_z\times(\cos\theta\,\hat{e}_z + \sin\theta\,\hat{e}_r)$

$$= \frac{\mu_0 m}{4\pi r^2}\sin\theta\,\hat{e}_\varphi \rightarrow \vec{B} = \nabla\times\vec{A} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta)$$

Ogólniejszy wzór: $\vec{B}_{dip} = \frac{\mu_0}{4\pi r^3}[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}]$

5) $\uparrow\vec H$ $\quad \vec H = H_0 \hat e_z \to \vec B = \mu_0 H_0 \hat e_z$ sztuczka: umiemy to rozwiązać dla potencjału



Niech $\vec B = \vec\nabla\varphi$, wtedy $\vec\nabla\cdot\vec B = \Delta\varphi = 0$

$$\varphi_\infty = \mu_0 H_0 z = \mu_0 H_0 r\cos\theta$$

$$\varphi = \begin{cases} \sum_{m=0}^{\infty} A_m r^m P_m(\cos\theta) & r<R \\ \sum_{m=0}^{\infty} C_m r^{-(m+1)} P_m(\cos\theta) + \mu_0 H_0 r\cdot\cos\theta & r>R \end{cases}$$

— bez C_m aby $\varphi(r=0)\neq\infty$

warunki brzegowe:

$\mathrm{I} \to \varphi_I(r=R) = \varphi_{II}(r=R) \to A_m R^m = C_m R^{-m-1}$, gdy $m\neq1$

$$A_1 R = \frac{C_1}{R^2} + \mu_0 H_0 R \to C_1 = (A_1 - \mu_0 H_0)R^3$$

$\underline{\mathrm{II}} \to \mu\, \partial_r \varphi_I(r=R) = \partial_r \varphi_{II}(r=R) \to \mu\cdot m A_m R^{m-1} = -(m+1) C_m R^{-(m+2)}$ gdy $m\neq1$

$$\mu A_1 = -2 C_1 R^{-3} + \mu_0 H_0$$

z zestawienia $A_m = C_m = 0$ dla $m\neq1$

$$\mu A_1 = \mu_0 H_0 - 2A_1 + 2\mu_0 H_0 \to A_1 = \mu_0 H_0 \frac{3}{\mu+2} \to C_1 = \mu_0 H_0 R^3 \frac{1-\mu}{2+\mu}$$

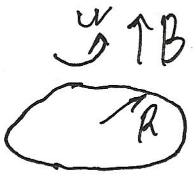
$$\varphi = \begin{cases} \mu_0 H_0 \frac{3}{\mu+2} r\cos\theta & r<R = \mu_0 H_0 \frac{3}{\mu+2} z \\ \mu_0 H_0 r\cos\theta + \frac{\mu_0 H_0 R^3}{r^2}\frac{1-\mu}{2+\mu}\cos\theta & r>R \end{cases}$$

$$\vec B = \vec\nabla\varphi = \hat e_r \partial_r \varphi + \frac{\hat e_\theta}{r}\partial_\theta\varphi + \frac{\hat e_\varphi}{r\sin\theta}\partial_\varphi\varphi = \begin{cases} \mu_0 H_0 \frac{3}{\mu+2}(\cos\theta\,\hat e_r - \sin\theta\,\hat e_\theta) \\ \mu_0 H_0(\cos\theta\,\hat e_r - \sin\theta\,\hat e_\theta) + \\ + \frac{\mu_0 H_0 R^3(1-\mu)}{2+\mu}\left[\frac{-2\cos\theta\,\hat e_r}{r^3} - \frac{\sin\theta\,\hat e_\theta}{r^3}\right] \end{cases}$$

przepraszam, kolizja oznaczeń, i tak $=0$

$\cancel{\vec H = \{\mu_0\mu}$

$$\vec H = \begin{cases} \frac{B}{\mu_0\mu} & r<R \\ \frac{B}{\mu_0} & r>R \end{cases}$$

6) 

$F_{el} = ma \rightarrow dQ \cdot E = dm \cdot \partial_t (\omega r)$

$dQ = \sigma\, dS$, $dm = \rho\, dS$ $\quad (\pi R^2 \rho = M,\ \pi R^2 \sigma = Q)$

M, Q

$$\int_0^{t_0} dt \int_0^R dr \int_0^{2\pi} d\varphi\; r \cdot \sigma E = \int_0^{t_0} dt \int_0^R dr \int_0^{2\pi} d\varphi\; r \cdot \rho r\, \partial_t \omega$$

L R

$$R = 2\pi\rho \int_0^{t_0} dt \int_0^R dr\; r^2 \cdot \partial_t \omega = 2\pi \rho \cdot \omega_0 \cdot \frac{R^3}{3} = \frac{2M\omega_0 R}{3}$$

$$L = \sigma \int_0^{t_0} dt \int_0^R dr \left[\int_0^{2\pi} d\varphi\; r \cdot E \right] = \sigma \int_0^{t_0} dt \int_0^R dr \oint_{r'=r} \vec{E}\, d\vec{r}' = \sigma \int_0^{t_0} dt \int_0^R dr \times$$

$$\times \left(- \iint_{r'=r} \partial_t \vec{B} \cdot d\vec{S} \right) = -\sigma \int_0^{t_0} dt \int_0^R dr \int_0^r dr' \int_0^{2\pi} d\varphi\; r'\, \partial_t B = -\sigma \cdot B_0 \times$$

$$\times 2\pi \int_0^R dr\, \frac{r^2}{2} = -2\pi \sigma B_0 \frac{R^3}{6} = \frac{2M\omega_0 R}{3}$$

$$-\frac{Q B_0 R}{\cancel{3}} = \frac{2 M \omega_0 R}{\cancel{3}} \rightarrow \omega_0 = -\frac{Q B_0}{2M}$$

(minus oznacza obrót zgodny ze wskazówką zegara)

seria 7