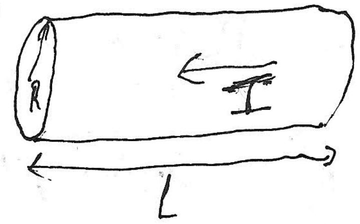
seria 8

1) 

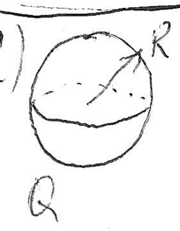
$\leftarrow \hat{z}$

na długość L spadek napięcia U

$$\vec{E} = \frac{U}{L}\hat{e}_z ; \text{ (z Ampera) } \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{e}_\varphi$$

$$\vec{P} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{U\mu_0 I}{2\pi r L \mu_0}\hat{e}_z \times \hat{e}_\varphi = \frac{-UI\hat{e}_r}{2\pi r L} ; \iiint \frac{\partial w}{\partial t} dV = -\oint \vec{P} d\vec{s}$$

$$\vec{n}_{podst} = \pm \hat{e}_z \rightarrow \iint_{podst.} \vec{P} d\vec{s} = 0 \rightarrow -\oint \vec{P} d\vec{s} = -\int_0^L dz \int_0^{2\pi} d\varphi\, r \frac{-UI}{2\pi r L} = \boxed{UI = \partial_t W}$$

2) 

Z Gaussa: $\vec{E} = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{e}_r & r \geq R \quad \left(4\pi r^2 E_r = \frac{Q}{\varepsilon_0}\right) \\ \frac{Q r}{4\pi\varepsilon_0 R^3}\hat{e}_r & r < R \quad \left(4\pi r^2 E_r = \frac{\rho}{\varepsilon_0}\cdot\frac{4}{3}\pi r^3 = \frac{Q r^3}{\varepsilon_0 R^3}\right)\end{cases}$

nie ma pola magnetycznego, więc $\vec{F} = \oint \overline{\overline{T}}_{el} d\vec{s}$; $(\overline{\overline{T}}_{el})_{ij} = \varepsilon_0 E_i E_j - \frac{\varepsilon_0}{2}\delta_{ij}|E|^2$

$$\oint_{półkula} \overline{\overline{T}}_{el} d\vec{s} = \left(\iint_{półsfera} + \iint_{dysk}\right)\overline{\overline{T}} \cdot d\vec{s} = \int_0^{2\pi} r\,d\varphi \int_0^{\pi/2} r \sin\theta\, d\theta \Big|_{r=R} \overline{\overline{T}} \cdot \hat{e}_r + \int_0^{2\pi} r\,d\varphi \sin\theta \int_0^R dr \Big|_{\theta=\frac{\pi}{2}} \overline{\overline{T}} \cdot (-\hat{e}_z)$$

Zwracamy uwagę na symetrię, jedyna możliwa siła $\vec{F} = F\hat{e}_z$

$$T_{zz}(\text{dysk}) = \frac{\varepsilon_0}{2}(E_z^2 - E_x^2 - E_y^2) = \frac{\varepsilon_0 Q^2 r^2}{(4\pi\varepsilon_0)^2 2 R^6}(\cos^2\theta - \sin^2\theta) = \frac{r^2 Q^2 \cos 2\theta}{32\pi^2\varepsilon_0 R^6} \qquad \hat{e}_r = \begin{pmatrix}\sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta\end{pmatrix}$$

$$T_{zz}\cdot(-\hat{e}_z)(\text{dysk}) = -\frac{Q^2\cos 2\theta}{32\pi^2\varepsilon_0 R^6} r^3 \sin\theta\, dr\, d\varphi \rightarrow F_d = \int_0^R dr \int_0^{2\pi} d\varphi \Big|_{\theta=\frac{\pi}{2}} -\frac{Q^2 r^3 \cos(2\theta)}{32\pi^2\varepsilon_0 R^6}\cdot\sin^2\theta = \frac{Q^2}{64\pi\varepsilon_0 R^2}$$

$$T_{zx}(\text{półsf.}) = \varepsilon_0 E_x E_z = \frac{\varepsilon_0 Q^2}{16\pi^2\varepsilon_0^2 R^4}\sin\theta\cos\theta\cos\varphi$$

$$T_{zy} = \frac{\varepsilon_0 Q^2}{16\pi^2\varepsilon_0^2 R^4}\sin\theta\cos\theta\sin\varphi \qquad \hat{e}_r = \begin{pmatrix}\sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta\end{pmatrix}$$

$$T_{zz} = \frac{\varepsilon_0 Q^2}{2\cdot 16\pi^2\varepsilon_0^2 R^4}(\cos^2\theta - \sin^2\theta)$$

$$\overline{\overline{T}}\cdot\hat{e}_r(\text{półsfera}) = \frac{Q^2}{16\pi^2\varepsilon_0 R^4}\left(\sin^2\theta\cos\theta\cos^2\varphi + \sin^2\theta\cos\theta\sin^2\varphi + \frac{\cos^3\theta}{2} - \frac{\cos\theta\sin^2\theta}{2}\right) =$$

$$= \frac{Q^2}{32\pi^2\varepsilon_0 R^4}\cos\theta \rightarrow \int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta \frac{Q^2}{32\pi^2\varepsilon_0 R^4} R^2 \underbrace{\sin\theta\cos\theta}_{\frac{1}{2}\sin 2\theta} = \frac{Q^2}{32\pi\varepsilon_0 R^2} = F_p$$

$$\boxed{\vec{F} = \frac{3Q^2}{64\pi\varepsilon_0 R^2}\hat{e}_z}$$

3) $\varphi = 0$, $\vec A = \vec A_0 f(\omega t - \vec k\cdot\vec r)$, dla uproszczenia $\vec A = \vec A_0 f(\omega t - kz)$

$\vec\nabla\cdot\vec A = A_{0z}\cdot(-kf') = 0 \to A_{0z} = 0 \to \vec A_0 \perp \vec k$, $\vec A_0\cdot\vec k = 0$

$\boxed{\begin{aligned}\partial_t f &= \omega f' \\ \partial_z f &= -kf'\end{aligned}}$

$\vec E = -\partial_t \vec A = -\omega \vec A_0 f'$; $\vec B = \vec\nabla\times\vec A = -kf'(-A_y\hat e_x + A_x\hat e_y) =$

$= -[\vec k\times\vec A_0]f'$;

digress: $A\times(B\times C) = \varepsilon_{ijk}\hat e_i A_j \varepsilon_{klm} B_l C_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\hat e_i A_j B_l C_m =$

$= \hat e_i B_i A_j C_j - \hat e_i C_i A_j B_j = \vec B(\vec A\cdot\vec C) - \vec C(\vec A\cdot\vec B)$

$$\vec S = \frac{\vec E\times\vec B}{\mu_0} = \frac{\omega\,\cancel{k}}{\mu_0} f'^2\, \vec A_0\times(\vec k\times\vec A_0) = \frac{\omega f'^2}{\mu_0}\vec k|A_0|^2$$

$$\rho_{en} = \frac{\varepsilon_0}{2}\vec E^2 + \frac{1}{2\mu_0}\vec B^2 = |A_0|^2 f'^2\left(\frac{\varepsilon_0\omega^2}{2} + \frac{|k|^2}{2\mu_0}\right) \overset{k^2=\varepsilon_0\mu_0\omega^2}{=\!=} \varepsilon_0\omega^2|A_0|^2 f'^2$$

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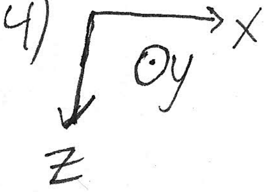
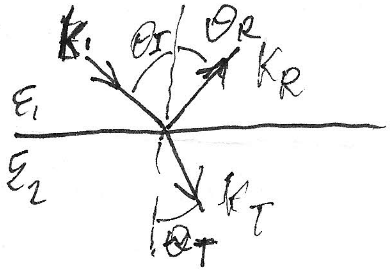
4)





$\vec\nabla\times(\vec\nabla\times\vec E) = -\partial_t\vec\nabla\times\vec B = -\mu\mu_0\partial_t\vec\nabla\times\vec H = -\mu\mu_0\varepsilon\varepsilon_0\partial_t^2\vec E$

$\vec\nabla\times\vec\nabla\times\vec E = \vec\nabla(\underbrace{\vec\nabla\cdot\vec E}_{\text{zero bo nie ma ładunków}}) - \Delta\vec E$

$$\Delta\vec E - \mu\mu_0\varepsilon\varepsilon_0\partial_t^2\vec E = 0$$

standardowe rozwiązanie: $\vec E = \vec E_0 e^{i\vec k\cdot\vec r}e^{-i\omega t}$

$$\Delta\vec E = (\partial_x^2 + \partial_y^2 + \partial_z^2)\vec E = -(k_x^2 + k_y^2 + k_z^2)\vec E = -k^2\vec E$$

$$\Delta\vec E - \mu\mu_0\varepsilon\varepsilon_0\partial_t^2\vec E = (-k^2 + \omega^2\mu\mu_0\varepsilon\varepsilon_0)\vec E = 0 \to k^2 = \omega^2\mu\mu_0\varepsilon\varepsilon_0$$

prędkość światła w próżni: $\mu_0\varepsilon_0 = \frac{1}{c^2}$; wsp. załamania $n^2 = \varepsilon\mu$

prędkość światła w ośrodku: $\mu\mu_0\varepsilon\varepsilon_0 = \frac{1}{v^2} = \frac{n^2}{c^2}$

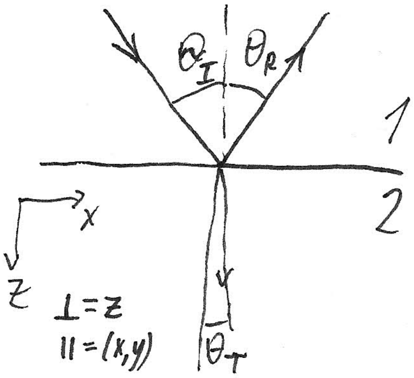
Związek dyspersyjny: $|k| = \frac{n\omega}{c}$

$$\vec\nabla\times\vec E = \varepsilon_{ijk}\hat e_i(\partial_j e^{i\vec k\cdot\vec r})E_{0k}e^{-i\omega t} = \varepsilon_{ijk}\hat e_i\cdot i k_j E_{0k}e^{i\vec k\vec r}e^{-i\omega t} =$$

$$= i\vec k\times\vec E = -\partial_t\vec B \longrightarrow \vec B = \vec B_0 e^{i\vec k\cdot\vec r}e^{-i\omega t} = \frac{\vec k\times\vec E_0}{\omega}e^{i\vec k\cdot\vec r}e^{-i\omega t}$$

Z formy $\vec{B}$ i z $\vec{\nabla}\cdot\vec{E}=0$ wynika, że $\vec{k}\perp\vec{E}_0\perp\vec{B}_0\perp\vec{k}$

puszczamy falę padającą w płaszczyźnie xz $(k_{yI}=0)$



wtedy mamy 3 fale: padającą

$$\vec{E}_I=\vec{E}_{I0}\,e^{i\vec{k}_I\cdot\vec{r}}e^{-i\omega_I t},\quad \vec{B}_I=\frac{\vec{k}_I\times\vec{E}_{I0}}{\omega}e^{i\vec{k}_I\cdot\vec{r}}e^{-i\omega_I t}$$

i analogiczne fale odbitą E_R i załamaną E_T

Warunki brzegowe (brak ładunków/prądów swobod.)

$$\begin{cases} E_{1\parallel}=E_{2\parallel}\\ D_{1\perp}=D_{2\perp}\\ H_{1\parallel}=H_{2\parallel}\\ B_{1\perp}=B_{2\perp}\end{cases}\longrightarrow\begin{cases} E_{Ix}+E_{Rx}=E_{Tx}\\ E_{Iy}+E_{Ry}=E_{Ty}\\ \varepsilon_1 E_{Iz}+\varepsilon_1 E_{Rz}=\varepsilon_2 E_{Tz}\\ \dfrac{B_{Ix}+B_{Rx}}{\mu_1}=\dfrac{B_{Tx}}{\mu_2};\quad \dfrac{B_{Iy}+B_{Ry}}{\mu_1}=\dfrac{B_{Ty}}{\mu_2}\\ B_{Iz}+B_{Rz}=B_{Tz}\end{cases}$$

$$\hat{k}_I=\sin\theta_I\,\hat{e}_x+\cos\theta_I\,\hat{e}_z$$
$$\hat{k}_R=\sin\theta_R\,\hat{e}_x-\cos\theta_R\,\hat{e}_z$$
$$\hat{k}_T=\sin\theta_T\,\hat{e}_x+\cos\theta_T\,\hat{e}_z$$

powinniśmy spełniać warunki brzegowe dla każdego t,x,y

dla każdego $t\to e^{-i\omega_I t}=e^{-i\omega_R t}=e^{-i\omega_T t}\to\omega_I=\omega_R=\omega_T=\omega$

(częstość fali nie zmienia się)

dla każdego $y\to e^{ik_{Iy}y}=e^{ik_{Ry}y}=e^{ik_{Ty}y}=0\to$ odbita i załamana fala są w tej samej płaszczyźnie

dla każdego $x\to e^{ik_{Ix}x}=e^{ik_{Rx}x}=e^{ik_{Tx}x}\to\frac{n_1\omega}{c}\sin\theta_I=\frac{n_1\omega}{c}\sin\theta_R=\frac{n_2\omega}{c}\sin\theta_T$

2 wnioski: $\to\theta_I=\theta_R$ (kąt odbicia równy kątowi padania)

$\to n_I\sin\theta_I=n_2\sin\theta_T$ (kąt załamania - prawo Snella)

dalej możemy oddzielnie rozpatrywać 2 polaryzacje:

TE $\to\vec{E}_0=E_0\hat{e}_y\to\vec{B}_0=\frac{n}{c}E_0(\sin\theta\,\hat{e}_z-\cos\theta\,\hat{e}_x)$ (+ dla $\vec{B}_R$)

TM $\to\vec{B}_0=B_0\hat{e}_y=\frac{n}{c}E_0\hat{e}_y\to\vec{E}_0=E_0(\cos\theta\,\hat{e}_x-\sin\theta\,\hat{e}_z)$ (- dla $\vec{E}_R$)

dla TE: $E_{Iy} + E_{Ry} = E_{Ty} \rightarrow E_I + E_R = E_T$

dla uproszczenia $\mu_1 = \mu_2 = 1$

$$B_{Iz} + B_{Rz} = B_{Tz} \rightarrow \frac{n_1}{c} E_I \sin\theta_I + \frac{n_1}{c} E_R \sin\theta_I = \frac{n_2}{c} E_T \sin\theta_T$$

$$n_1 \sin\theta_I (E_I + E_R) = n_2 \sin\theta_T E_T,$$ nic nowego

$$\frac{B_{Ix} + B_{Rx}}{\mu_1} = \frac{B_{Tx}}{\mu_2} \rightarrow n_1 \cos\theta_I (E_I - E_R) = n_2 \cos\theta_T (E_R + E_I)$$

$$E_R (n_1 \cos\theta_I + n_2 \cos\theta_T) = E_I (-n_2 \cos\theta_T + n_1 \cos\theta_I)$$

$$R = \left|\frac{E_R}{E_I}\right|^2 = \left(\frac{n_1 \cos\theta_I - n_2 \cos\theta_T}{n_1 \cos\theta_I + n_2 \cos\theta_T}\right)^2 !$$

dla TM: $B_{Iy} + B_{Ry} = B_{Ty} \rightarrow n_1 (E_I + E_R) = n_2 E_T$

$$\varepsilon_1 E_{Iz} + \varepsilon_1 E_{Rz} = \varepsilon_2 E_{Tz} \rightarrow \varepsilon_1 \sin\theta_I \cancel{n_2} (E_I + E_R) = \varepsilon_2 \sin\theta_T \cancel{n_2} E_T$$

nic nowego

$$E_{Ix} + E_{Rx} = E_{Tx} \rightarrow \cos\theta_I (E_I - E_R) = \cos\theta_T E_T = \cos\theta_T \cdot \frac{n_1}{n_2} (E_I + E_R)$$

$$E_I (n_2 \cos\theta_I - n_1 \cos\theta_T) = E_R (n_2 \cos\theta_I + n_1 \cos\theta_T)$$

$$R = \left|\frac{E_R}{E_I}\right|^2 = \left(\frac{n_2 \cos\theta_I - n_1 \cos\theta_T}{n_2 \cos\theta_I + n_1 \cos\theta_T}\right)^2 !$$

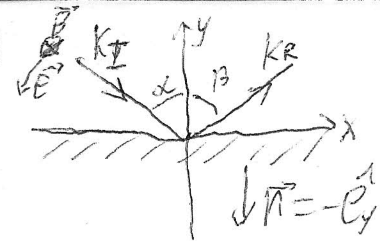
kąt Brewstera: $R_{TM} = 0 \rightarrow n_2^2 \cos^2\theta_I = n_1^2 \cos^2\theta_T$

$$\cos^2\theta_T \cdot n_1^2 = n_1^2 - n_1^2 \sin^2\theta_T = n_1^2 - \frac{n_1^2}{n_2^2} \cdot n_1^2 \sin^2\theta_I = n_2^2 \cos^2\theta_I$$

$$\left(n_1^2 - \frac{n_1^4}{n_2^2}\right) \sin^2\theta_I = (n_2^2 - n_1^2) \cos^2\theta_I \rightarrow \underbrace{\operatorname{tg}\theta_I = \frac{(n_2^2 - n_1^2) n_2^2}{(n_2^2 - n_1^2) n_1^2} = \frac{n_2}{n_1}}$$

ciekawostka: przy kącie Brewstera $\theta_R + \theta_T = \frac{\pi}{2}$

ciekawostka 2: $T = \left|\frac{E_T}{E_I}\right|^2 = 1 - R$ niezależnie od polaryzacji

5) 

polaryzacja TM: $\vec{B}_0 = B_0 \hat{e}_z$

warunek brzegowy w metalu: $\vec{E} = 0$, $\vec{B} = 0$

spełnione dla każdego t, x, z: częstość fali taka sama, kąty padania i odbicia równe, zachowana polaryzacja

$$\vec{E}_I = E_I e^{i\vec{k}_I \vec{r} - i\omega t}(-\cos\theta\, \hat{e}_x - \sin\theta\, \hat{e}_y), \quad \hat{K}_I = \sin\theta\, \hat{e}_x - \cos\theta\, \hat{e}_y$$

$$\vec{B}_I = \frac{n}{c} E_I e^{i\vec{k}_I \vec{r} - i\omega t} \hat{e}_z$$

$$\vec{E}_R = E_R e^{i\vec{k}_R \vec{r} - i\omega t}(\cos\theta\, \hat{e}_x - \sin\theta\, \hat{e}_y), \quad \hat{K}_R = \sin\theta\, \hat{e}_x + \cos\theta\, \hat{e}_y, \quad \vec{B}_R = \frac{-n}{c} E_R e^{i\vec{k}_R \vec{r} - i\omega t} \hat{e}_z$$

warunki zszycia: $\begin{cases} E_\parallel = 0 & B_\perp = 0 \\ D_\perp = \sigma & H_\parallel = (K \times n) \end{cases} \rightarrow \begin{cases} E_{Ix} + E_{Rx} = 0 \rightarrow E_I = E_R \\ \varepsilon_1 (E_{Iy} + E_{Ry}) = \sigma \\ (\vec{K} \times \vec{n}) \cdot \hat{e}_x = 0 \rightarrow \vec{K} = K \hat{e}_x \\ (\vec{K} \times \vec{n}) \cdot \hat{e}_z = \frac{+1}{\mu_1}(B_{Iz} + B_{Rz}) \end{cases}$

$$\begin{cases} \varepsilon_1 \cdot -2 E_I \sin\theta = \sigma \\ -2 \frac{E_I n}{\mu_1 c} = K_x \end{cases}$$

$\rightarrow$ sprawdzamy zachowanie ładunku

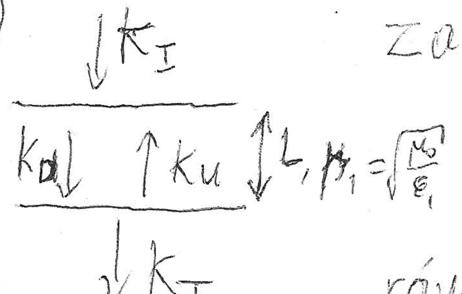
$$\partial_t \sigma = 2i\omega\, \varepsilon_1 E_I \sin\theta, \quad \vec{\nabla} \cdot \vec{K} = \partial_x K_x = 2i k_x \frac{E_I n}{\mu_1 c}$$

$$\partial_t \sigma + \vec{\nabla} \cdot \vec{K} = 2i E_1 \sin\theta (\omega \varepsilon_1 \mu_1 \varepsilon_0 \mu_0 - k) = 0$$

związek dyspersyjny

$$\vec{S}_I = \frac{\vec{E}_I \times \vec{B}_I}{\mu_1 \mu_0} = \frac{n}{\mu_1 c} E_I^2 (e^{i\vec{k}_I \vec{r} - i\omega t})^2 \cdot \hat{k}_I; \quad \vec{S}_R = \frac{n}{\mu_1 c} E_I^2 (e^{i\vec{k}_R \vec{r} - i\omega t})^2 \cdot \hat{k}_R$$

sumaryczny $\vec{S}$ ma kierunek $\hat{e}_x$ – metal jest lustrem

6) 

załóżmy polaryzację: $\vec{E} = E_0 \hat{e}_y e^{ikx} e^{-i\omega t}$

dla ośrodka $k = \frac{2\pi n}{\lambda}$, w próżni $k_0 = \frac{2\pi}{\lambda}$

równania: warunki zszycia + zachowanie energii

$$\begin{cases} E_I = E_d + E_u \\ E_d e^{\frac{2\pi i n d}{\lambda}} + E_u e^{-\frac{2\pi i n d}{\lambda}} = E_T e^{\frac{2\pi i d}{\lambda}} \\ |E_I|^2 = |E_T|^2 \end{cases}$$

$$\left(E_d e^{\frac{2\pi i n d}{\lambda}} + E_u e^{\frac{-2\pi i n d}{\lambda}}\right)\left(E_d e^{\frac{-2\pi i n d}{\lambda}} + E_u e^{\frac{2\pi i n d}{\lambda}}\right) = |E_T|^2 = |E_I|^2 = (E_d + E_u)^2$$

$$E_d^2 + E_u^2 + E_d E_u \left(e^{\frac{4\pi i n d}{\lambda}} + e^{-\frac{4\pi i n d}{\lambda}}\right) = E_d^2 + E_u^2 + 2 E_d E_u$$

$$\frac{e^{\frac{4\pi i n d}{\lambda}} + e^{-\frac{4\pi i n d}{\lambda}}}{2} = \cos\left[\frac{4\pi n d}{\lambda}\right] = 1 \rightarrow \frac{2nd}{\lambda} = m \in \mathbb{C} \rightarrow nd = m\frac{\lambda}{2}$$

powinno mieścić się dokładnie m połówek długości fali, czyli w środku ma być fala stojąca!

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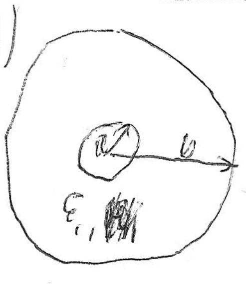
1) 

seria 9

z Gaussa: $\iint \vec{D} d\vec{s} = 2\pi r \varepsilon_0 \varepsilon E = Q$

$E = \frac{Q}{2\pi\varepsilon\varepsilon_0}\frac{1}{r} \rightarrow U_A - U_B = \frac{Q}{2\pi\varepsilon\varepsilon_0} \ln\frac{b}{a} \rightarrow C = \frac{Q}{U} = \frac{2\pi\varepsilon\varepsilon_0}{\ln b/a}$

z Ampera: $\oint \vec{B} d\vec{l} = 2\pi r B_\varphi = \mu_0 \mu_1 I$

$B = \frac{\mu_0 \mu_1 I}{2\pi r} \rightarrow \Phi = \iint \vec{B} d\vec{s} = \frac{\mu_0 \mu_1 I}{2\pi} \ln\frac{b}{a} \rightarrow L = \frac{\Phi}{I} = \frac{\mu_0 \mu}{2\pi} \ln\frac{b}{a}$

$L \cdot C = \varepsilon_0 \mu_0 \varepsilon \mu = \frac{n^2}{c^2}$

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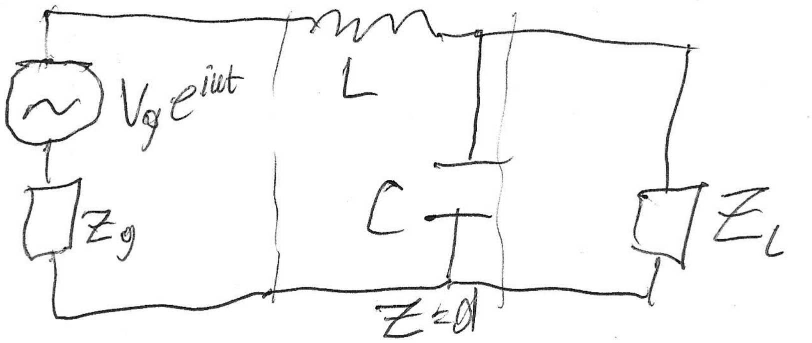
2)



równania telegrafisty:

$$-\partial_z U = L\partial_t I$$
$$-\partial_z I = C\partial_t U$$

$$\partial_z\partial_t U = -L\partial_z\partial_t I = -L\partial_t \cdot C\partial_t U$$

$\partial_z^2 U = -LC\partial_t^2 U$, wygląda jak równanie falowe

$$U = (\underbrace{U_+ e^{ikz}}_{\text{fala w prawo}} + \underbrace{U_- e^{-ikz}}_{\text{fala w lewo}}) e^{i\omega t}$$

(brak $e^{-i\omega t}$, bo mamy taki generator)

$$+\partial_z^2 U = -(U_+ e^{ikz} + U_- e^{-ikz})e^{i\omega t}\cdot k^2 = -LC\omega^2 (U_+ e^{ikz} + U_- e^{-ikz})e^{i\omega t}$$

związek dyspersyjny: $k^2 = LC\omega^2 \to k = \sqrt{LC}\,\omega$

$$-\partial_z U = (-ikU_+ e^{ikz} + ikU_- e^{-ikz})e^{i\omega t} = i\omega L(I_+ e^{ikz} + I_- e^{-ikz})e^{i\omega t}$$

$\begin{cases} I_+ = -\sqrt{\frac{C}{L}}U_+ \\ I_- = \sqrt{\frac{C}{L}}U_- \end{cases}$; zostały tylko warunki brzegowe

$$\begin{cases} U(z=0) = V_g e^{i\omega t} - Z_g I(z=0) \\ U(z=d) = I(z=d)\cdot Z_L \end{cases} \to \begin{cases} U_+ + U_- = V_g - Z_g\left(\sqrt{\frac{C}{L}}U_- - \sqrt{\frac{C}{L}}U_+\right) \\ U_+ e^{ikd} + U_- e^{-ikd} = Z_L\sqrt{\frac{C}{L}}\left(-U_+ e^{ikd} + U_- e^{-ikd}\right) \end{cases}$$

$$U_+\left(e^{ikd} + Z_L\sqrt{\frac{C}{L}}e^{ikd}\right) = U_-\left(Z_L\sqrt{\frac{C}{L}}e^{-ikd} - e^{-ikd}\right) \to U_- = U_+ e^{2ikd}\frac{Z_L\sqrt{\frac{C}{L}}+1}{Z_L\sqrt{\frac{C}{L}}-1}$$

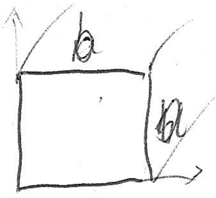
$$U_+\left(1-\sqrt{\frac{C}{L}}Z_g\right) + U_+ e^{2ikd}\frac{Z_L\sqrt{\frac{C}{L}}+1}{Z_L\sqrt{\frac{C}{L}}-1}\left(1+\sqrt{\frac{C}{L}}Z_g\right) = V_g$$

$$U_+ = \frac{V_g}{1-\sqrt{\frac{C}{L}}Z_g + e^{2ikd}\frac{\left(Z_L\sqrt{\frac{C}{L}}+1\right)\left(Z_g\sqrt{\frac{C}{L}}+1\right)}{Z_L\sqrt{\frac{C}{L}}-1}}$$

$Z_L = 0 \to U(z=d) = 0 \to U_- = -U_+ e^{2ikd}$

$Z_L = \infty \to I(z=d) = 0 \to U_- = U_+ e^{2ikd}$

IX 3)



metal: $E_{\parallel}=0$, $B_{\perp}=0$

$\vec{E}=\vec{E}_0 e^{ikz-i\omega t}$, $\vec{B}=\vec{B}_0 e^{ikz-i\omega t}$

$$\begin{vmatrix} e_x & e_y & e_z \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\begin{cases} \vec{\nabla}\times\vec{E}=-\partial_t\vec{B} \\ \vec{\nabla}\times\vec{B}=\frac{1}{c^2}\partial_t\vec{E} \leftrightarrow \vec{\nabla}\times\vec{H}\cdot\mu_0=\mu_0\,\partial_t D \end{cases} \to \begin{cases} \partial_x E_y-\partial_y E_x=i\omega B_z \\ \partial_y E_z-ikE_y=i\omega B_x \\ ikE_x-\partial_x E_z=i\omega B_y \end{cases}$$

$\partial_x B_y-\partial_y B_x=-\frac{i\omega}{c^2}E_z$

$\partial_y B_z-ikB_y=-\frac{i\omega}{c^2}E_x$

$ikB_x-\partial_x B_z=-\frac{i\omega}{c^2}E_y$

$$\frac{i}{\frac{\omega^2}{c^2}-k^2}=M$$

$$\begin{cases} E_x=M(k\partial_x E_z+\omega\partial_y B_z) \\ E_y=M(k\partial_y E_z-\omega\partial_x B_z) \end{cases} \quad B_x=M\left(k\partial_x B_z-\frac{\omega}{c^2}\partial_y E_z\right) \quad B_y=M\left(k\partial_y B_z+\frac{\omega}{c^2}\partial_x E_z\right)$$

$$\partial_x E_y-\partial_y E_x=M(k\partial_x\partial_y E_z+\omega\partial_x^2 B_z-k\partial_y\partial_x E_z+\omega\partial_y^2 B_z)=i\omega B_z$$

$$\left[\partial_x^2+\partial_y^2+\frac{\omega^2}{c^2}-k^2\right]B_z=0 \xleftrightarrow{\text{analog}} \left[\partial_x^2+\partial_y^2+\frac{\omega^2}{c^2}-k^2\right]E_z=0$$

TE: $E_z=0$, $B_z=B_0\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$

TM: $B_z=0 \to \left(\partial_x^2+\partial_y^2+\frac{\omega^2}{c^2}-k^2\right)E_z=0$

$$E_z=\left[A_x\sin(k_x x)+B_x\cos(k_x x)\right]\left[A_y\sin(k_y y)+B_y\cos(k_y y)\right]$$

$x=0$, $y=0$ $\Rightarrow E_z=0 \to B_x=B_y=0 \to E_z=A\sin(k_x x)\sin(k_y y)=E_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$

$x=a$, $y=b$ $\to E_z=0 \Rightarrow \begin{matrix}\sin(k_x a)=0 \\ \sin(k_y b)=0\end{matrix} \to \begin{matrix}k_x=m\pi/a \\ k_y=n\pi/b\end{matrix}$, $m,n\in$ int $\to k^2=\frac{\omega^2}{c^2}-\frac{m^2\pi^2}{a^2}-\frac{n^2\pi^2}{b^2}$

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4)

$x(t=0)=0 \to \vec{r}'=vt\hat{e}_x$

$(\vec{r}-\vec{r}')=\vec{r}-vt\hat{e}_x$

$|\vec{r}-\vec{r}'|^2_{ret}=r^2+v^2t_r^2-2r_x v t_r=c^2t^2-2c^2tt_r+c^2t_r^2$

$t_r^2(v^2-c^2)+t_r(-2r_x v+2c^2t)+r^2-c^2t^2=0$

$\Delta=4(c^2t-r_x v)^2-4(v^2-c^2)(r^2-t^2c^2)$

$t_r=\frac{2(\vec{r}\vec{v}-c^2t)\pm 2\sqrt{(c^2t-\vec{r}\vec{v})^2-(v^2-c^2)(r^2-t^2c^2)}}{2}$

(minus bo dla $v=0$ $t_r=t-\frac{r}{c}$)

$$t_r=\frac{c^2t-\vec{r}\vec{v}-\sqrt{(c^2t-\vec{r}\vec{v})^2+(c^2-v^2)(r^2-c^2t^2)}}{c^2-v^2} \to |r-r'|=c(t-t_r), \quad \widehat{r-r'}=\frac{\vec{r}-\vec{v}t_r}{c(t-t_r)}$$

$$|r-r'|\left(1-\frac{1}{c}\widehat{r-r'}\cdot\vec{v}\right)=ct-ct_r-\cancel{(t-t_r)}\frac{\vec{r}\vec{v}-v^2t_r}{c\cancel{(t-t_r)}}=ct-\frac{\vec{r}\vec{v}}{c}-\left(-\frac{v^2}{c}+c\right)t_r=$$

$$=ct-\frac{\vec{r}\vec{v}}{c}-\frac{1}{c}\left(c^2t-\vec{r}\vec{v}-\sqrt{\dots}\right)=\frac{1}{c}\sqrt{\dots}$$

$$V=\frac{q}{4\pi\varepsilon_0}\frac{c}{\sqrt{(c^2t-\vec{r}\vec{v})^2+(c^2-v^2)(r^2-c^2t^2)}}$$