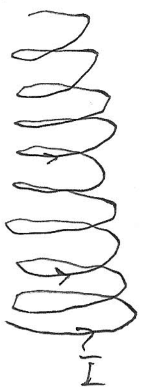
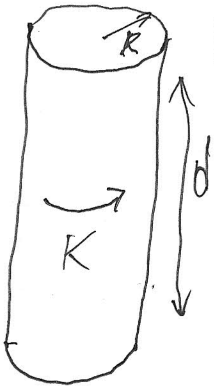
1)

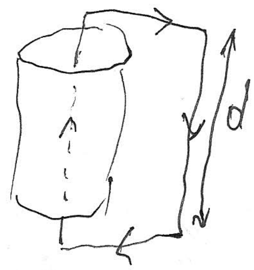


$\Rightarrow$



$\vec{K} = n \cdot \cancel{d} \cdot I \, \hat{e}_\varphi$

$\vec{B} = B \hat{e}_z$ i tylko dla $r < R$



$\oint \vec{B} d\vec{l} = B_z \cdot d$

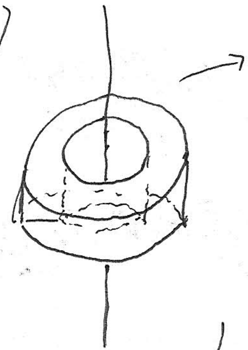
$\iint \mu_0 \vec{j} d\vec{s} = \mu_0 n \cancel{d} I \cdot d$

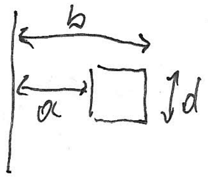
$B_z = \mu_0 I n \cancel{d}$

strumień przez n·d pierścieni

$\iint \vec{B} d\vec{s} = \mu_0 I n \cdot nd \cdot \pi R^2 \rightarrow \boxed{\frac{L}{d} = \frac{\iint \vec{B} d\vec{s}}{d \cdot I} = \mu_0 n^2 \cdot \pi R^2}$

2a)





$L_{12} = \frac{\iint \vec{B}_2 d\vec{s}_1}{I_2}$

1 – toroid

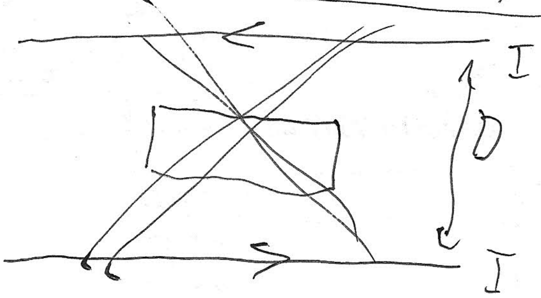
2 – drut (bo całka z pow. druta to magia)

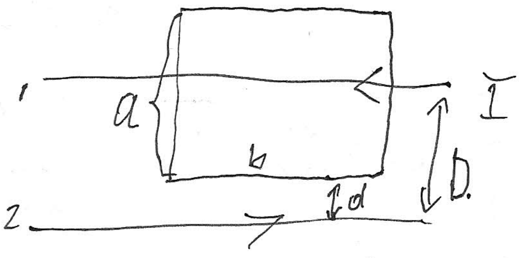
pole drutu: $2\pi r B_2 = \mu_0 I_2 \rightarrow B_2 = \frac{\mu_0 I_2}{2\pi r}$

bo przez N takich pierścieni

$L_{12} = \frac{\int_a^b dr \int_0^d dz \cdot \frac{\mu_0 \cancel{I_2}}{2\pi r}}{\cancel{I_2}} \cdot N = \frac{\mu_0 d N}{2\pi} \ln \frac{b}{a}$

2b)



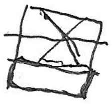


Pole od dwóch drutów: $B = B_1 + B_2 = \frac{\mu_0 I}{2\pi (D-r)} + \frac{\mu_0 I}{2\pi r}$

dla druta 1 zauważamy symetrię pola:

~~dla~~ ~~[illegible]~~ $B_1(D+dr) = -B_1(D-dr) \Rightarrow \int_0^b \int_{D-dr}^{D+dr} \vec{B}_1 d\vec{s} = 0 \Rightarrow \iint \vec{B}_1 d\vec{s}$

ma zmniejszone granice

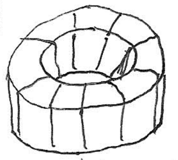
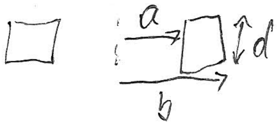


$; \; d+a = D+dr \rightarrow dr = d+a-D$

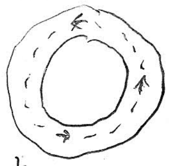
$D - dr = D-d-a+D = 2D-d-a$

$\iint \vec{B}_1 d\vec{s} = \frac{\mu_0 I}{2\pi} \int_0^b dz \int_d^{2D-d-a} dr \frac{1}{D-r} = \left| \begin{matrix} w = D-r \\ dw = -dr \end{matrix} \right| = \frac{\mu_0 I b}{2\pi} \int_{d+a-D}^{D-d} \frac{dw}{w} = \frac{\mu_0 I b}{2\pi} \ln \frac{D-d}{d+a-D}$

$\iint \vec{B}_2 d\vec{s} = \frac{\mu_0 I b}{2\pi} \ln \frac{d+D}{d} \rightarrow L = \frac{\iint \vec{B} d\vec{s}}{I} = \frac{\mu_0 b}{2\pi} \ln \frac{D^2-d^2}{d^2+ad-dD}$

3)  $\rightarrow$ 

N zwojów

Amper: 

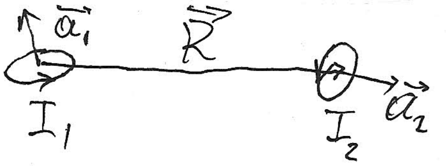
$2\pi r B_\varphi = \mu_0 N I$

$B = \frac{\mu_0 N I}{2\pi r}$

$$L = \frac{\iint \vec{B}\, d\vec{s}}{I} = \frac{N \cdot \int_0^d dz \int_a^b dr\, \frac{\mu_0 N I}{2\pi} \frac{1}{r}}{I} = \frac{\mu_0 N^2 d}{2\pi} \ln\frac{b}{a}$$

$$W = \frac{L I^2}{2} = \frac{\mu_0 I^2 N^2 d}{4\pi} \ln\frac{b}{a}$$

---

4) 

$\vec{B}_{dip} = \frac{\mu_0}{4\pi r^5}\left(3(\vec{m}\cdot\vec{r})\vec{r} - \vec{m}|r|^2\right)$, $\vec{m} = I \cdot \vec{a}$

R jest bardzo duże, więc pole przy całym dipolu jest takie samo

$$L = \frac{\iint \vec{B}_1 d\vec{s}_2}{I_1} = \frac{\mu_0}{4\pi R^5}\left[3(\vec{a}_1\cdot\vec{R})\cdot\vec{R} - \vec{a}_1|R|^2\right]\cdot\vec{a}_2 = \frac{\mu_0\left[3(a_1\cdot R)(a_2\cdot R) - (a_1\cdot a_2)(R\cdot R)\right]}{4\pi R^5}$$

$$\frac{\partial W}{\partial t} = -\mathcal{E}_2 \cdot I_1 = \partial_t (L\cdot I_2) I_1 \rightarrow W = L\cdot I_1 I_2 = \frac{\mu_0\left[3(\vec{m}_1\cdot\vec{R})(\vec{m}_2\cdot\vec{R}) - (\vec{m}_1\cdot\vec{m}_2)(\vec{R}\cdot\vec{R})\right]}{4\pi R^5}$$

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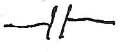
przypomnienie: prawa Kirchoffa

1 $\rightarrow$ w każdym węźle $\sum I_m = 0$

2 $\rightarrow$ w każdym oczku $\underbrace{\sum U}_{\text{spadki napięć}} = \sum \mathcal{E}$ $\leftarrow$ siła elektromotoryczna 

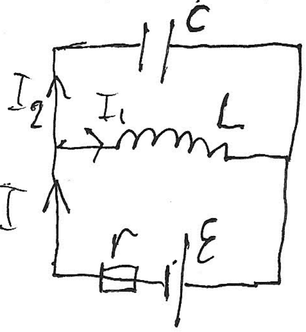
spadki napięć

 $U = I \cdot R$

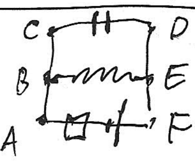
 $U = \frac{Q}{C} = \frac{\int I\, dt}{C}$

 $U = L \cdot \partial_t I$

---

5) 

$I = I_1 + I_2$



Oczka: BCDE
ACDF
ABEF

ACDF: $\frac{Q_2}{C} + Ir = \mathcal{E} \rightarrow Q_2 = -IrC + \mathcal{E}C$

BCDE: $\frac{Q_2}{C} - L\partial_t I_1 = 0$

ABEF: $Ir + \partial_t I_1 \cdot L = \mathcal{E}$

wystarczą 2 równania

$\partial_t Q_2 = I_2 = CL \cdot \partial_t^2 I_1$ (z BCDE)

$Ir + L\cdot\partial_t I_1 = I_1 r + rCL\cdot\partial_t^2 I_1 + L\cdot\partial_t I_1 = \mathcal{E}$

$$\partial_t^2 I_1 + \frac{1}{rC}\partial_t I_1 + \frac{1}{LC} I_1 = \frac{\mathcal{E}}{RLC}$$

najpierw równanie jednorodne:

$$\partial_t^2 I_1 + \frac{1}{rC}\partial_t I_1 + \frac{1}{LC} I_1 = 0 \rightarrow I_1 = A_1 e^{\lambda_1 t} + B_1 e^{\lambda_2 t}$$

$$\partial_t^2 (A e^{\lambda t}) + \frac{1}{rC}\partial_t (A e^{\lambda t}) + \frac{1}{LC} A e^{\lambda t} = A e^{\lambda t}\left(\lambda^2 + \frac{\lambda}{rC} + \frac{1}{LC}\right) = 0$$

$$\lambda^2 + \frac{1}{rC}\lambda + \frac{1}{LC} = 0 \rightarrow \Delta = \frac{1}{r^2C^2} - \frac{4}{LC} = \frac{1}{r^2C^2}\left(1 - \frac{4R^2C}{L}\right)$$

$$\lambda_{1,2} = \frac{-\frac{1}{rC} \pm \frac{1}{rC}\sqrt{1-\frac{4R^2C}{L}}}{2} = -\frac{1}{2rC}\left(1 \pm \sqrt{1-\frac{4R^2C}{L}}\right)$$

teraz niejednorodne: $I_1 = A_1 e^{\lambda_1 t} + B_1 e^{\lambda_2 t} + D_1$

$$\frac{D_1}{LC} = \frac{\mathcal{E}}{rLC} \rightarrow D_1 = \frac{\mathcal{E}}{r} \rightarrow I_1 = A_1 e^{\lambda_1 t} + B_1 e^{+\lambda_2 t} + \frac{\mathcal{E}}{r}$$

warunki początkowe: 1) brak prądu na starcie w cewce

$I_1(0) = 0,\ Q_2(0) = 0$ 2) rozładowany kondensator

$$1 \rightarrow A_1 + B_1 = -\frac{\mathcal{E}}{r} \rightarrow B_1 = -\left(\frac{\mathcal{E}}{r} + A_1\right)$$

$$2 \rightarrow Q_2 = LC\,\partial_t I_1 \text{ (z BCDE)} \rightarrow \partial_t I_1 = 0 \rightarrow \lambda_1 A_1 - \lambda_2 \frac{\mathcal{E}}{r} - \lambda_2 A_1 = 0$$

~~$A_1 = \lambda_1 - \lambda_2$~~

$$(\lambda_1 - \lambda_2) A_1 = \lambda_2 \frac{\mathcal{E}}{r} \rightarrow \boxed{A_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2}\frac{\mathcal{E}}{r}}$$ reszta do wyznacz.

Popatrzmy uważniej na prąd: $I_1 = A_1 e^{\lambda_1 t} - A_1 e^{\lambda_2 t} + \frac{\mathcal{E}}{r}(1 - e^{\lambda_2 t})$

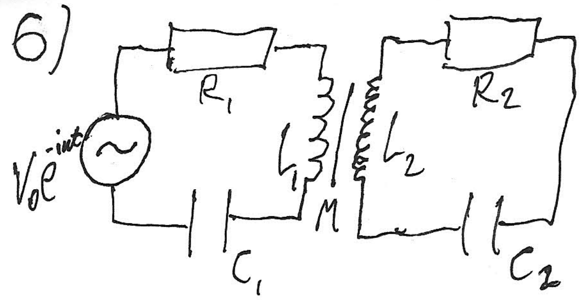
$$\left.\begin{array}{l} \lambda_1 = \frac{-1}{2rC} + \frac{\sqrt{1-\frac{4R^2C}{L}}}{2rC} < 0 \\ \lambda_2 = \frac{-1}{2rC} - \frac{\sqrt{1-\frac{4R^2C}{L}}}{2rC} < \lambda_1 < 0 \end{array}\right\} \rightarrow A_1 e^{\lambda_1 t} - A_1 e^{\lambda_2 t} = A_1 e^{-\frac{t}{2rC}}\left(e^{\frac{\sqrt{\dots}\,t}{2rC}} - e^{-\frac{\sqrt{\dots}\,t}{2rC}}\right) =$$

$$= A_1 e^{-\frac{t}{2rC}} \cdot 2\sinh\left[\frac{\sqrt{\dots}\,t}{2rC}\right]$$

czyli prąd ma zanikające oscylacje i prąd zbiegający do opornikowego

$$I = A \cdot e^{-\frac{t}{2rC}} \cdot 2\sinh\left[\frac{\sqrt{1-4R^2C/L}}{2rC}t\right] + \frac{\mathcal{E}}{r}\left[1 - e^{-\frac{\sqrt{1-4R^2C/L}}{2rC}t}\right]$$

6)



W zadaniu $C_1 = C_2 = R_1 = 0$, $M^2 = L_1 L_2$

M oznacza wzajemną indukcję $(U_1 = M\partial_t I_2)$

Lewe oczko: $I_1 R_1 + L_1 \partial_t I_1 + \frac{Q_1}{C_1} + M\partial_t I_2 = V_0 e^{-i\omega t}$

prawe oczko: $I_2 R_2 + L_2 \partial_t I_2 + \frac{Q_2}{C_2} + M\partial_t I_1 = 0$

Po podstawieniu R_1, C_1, C_2

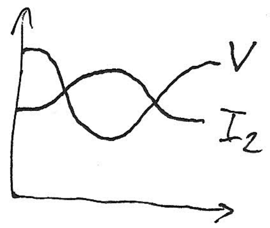
$$\begin{cases} L_1 \partial_t I_1 + M \partial_t I_2 = V_0 e^{-i\omega t} \\ L_2 \partial_t I_2 + M \partial_t I_1 + I_2 R_2 = 0 \end{cases} \longrightarrow \partial_t I_1 = \frac{V_0 e^{-i\omega t} - M\partial_t I_2}{L_1}$$

$$L_2 \partial_t I_2 + \frac{M V_0}{L_1} e^{-i\omega t} - \frac{M^2}{L_1}\partial_t I_2 + I_2 R_2 = 0$$

$$\Big(\partial_t I_2 - \underbrace{\frac{M^2}{L_1 L_2}\partial_t I_2}_{\text{przy } M^2 = L_1 \cdot L_2 \to 0}\Big) + \frac{M V_0}{L_1 L_2} e^{-i\omega t} + \frac{I_2 R_2}{L_2} = 0$$

$$I_2 = -\frac{M V_0}{L_1 R_2} e^{-i\omega t} = \underbrace{\frac{M V_0}{L_1 R_2}}_{\text{amplituda}} e^{-i\omega t} e^{i\pi}$$

faza – przesunięcie o 180°



czyli płynący prąd

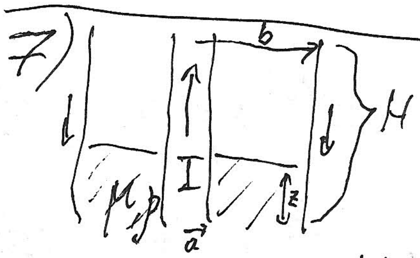
$$J_2 = \mathrm{Re}(I_2) = -\frac{M V_0}{L_1 R_2}\cos(\omega t)$$

$$P_2 = J_2^2 R_2 \text{ (z Joule'a)} = \frac{M^2 V_0^2}{L_1^2 R_2}\cos^2(\omega t) = \frac{L_2}{L_1 R_2} V_0^2 \cos^2(\omega t)$$

średnia moc na 1 okres: $\langle P_2 \rangle = \frac{1}{T}\int_0^T P\,dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{L_2 V_0^2}{L_1 R_2}\cos^2(\omega t)\,dt$

$$= \boxed{\frac{L_2 V_0^2}{2 L_1 R_2}}$$

7)



Z Ampera (uwaga, Amper w ośrodku $\to \oint \vec{H}\,d\vec{l} = \iint \vec{J}_{free}\,d\vec{S}$)

$$H = \frac{I}{2\pi r} \to B = \begin{cases} \frac{\mu_0 I}{2\pi r}, & \text{w powietrzu} \\ \frac{\mu\mu_0 I}{2\pi r}, & \text{w cieczy} \end{cases} \to W = \iiint \frac{\vec{B}\cdot\vec{H}}{2}\,dV$$

$$W_{MAG} = \int_a^b dr \int_0^{2\pi} r\,d\varphi \left[\int_0^z dz \frac{\mu\mu_0}{2}\left(\frac{I}{2\pi r}\right)^2 + \int_z^H dz \frac{\mu_0}{2}\left(\frac{I}{2\pi r}\right)^2\right] =$$

$$= \ln\frac{b}{a}\cdot 2\pi \cdot \frac{I^2}{4\pi^2}\cdot\frac{\mu_0}{2}(H + \mu' z) = \frac{\mu_0 I^2 \ln\frac{b}{a}}{4\pi}(H + \mu' z) \to F_{MAG} = \partial_z W_{MAG} = \frac{\mu_0 \mu' I^2}{4\pi}\ln\frac{b}{a}$$

$$W_{grav} = \rho V g \frac{z}{2} = \frac{\pi(b^2 - a^2)}{2}\rho g z^2 \to F_{grav} = \partial_z W_{grav} = \pi(b^2-a^2)\rho g z \quad \boxed{\mu' = \mu - 1}$$

przepraszam

$$F_g = F_M \to \boxed{z = \frac{\mu_0 \mu' I^2 \ln\frac{b}{a}}{4\pi^2 \rho g (b^2 - a^2)}}$$

ważne: μ może być mniejsze od 1, wtedy wypychamy