0.1 Pure math

Summary notation: if indices of at least two elements repeat - sum over. Example: $A_iB_jC_jD_kE_k = \sum_{j,k} A_iB_jC_jD_kE_k$

Kronecker delta: $\delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$

Levi-Civita symbol: $\epsilon_{ijk} = \begin{cases} 1, & \text{if } (i,j,k) \subseteq [(1,2,3),(2,3,1),(3,1,2)] \\ -1, & \text{if } (i,j,k) \subseteq [(1,3,2),(3,2,1),(2,1,3)] \\ 0, & \text{if } i = j \vee i = k \vee j = k \end{cases}$

$\epsilon_{ijk} = \epsilon_{jki} = \epsilon_{kij} = -\epsilon_{ikj}$
$\epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

Nabla operator: $\vec{\nabla} = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}$

Scalar multiplication: $\vec{A} \cdot \vec{B} = A_iB_i$; Vector multiplication: $\vec{A} \times \vec{B} = \epsilon_{ijk}\vec{e_i}A_jB_k$

Typical coordinates: cartesian (x,y,z); cylindrical (ρ,ϕ,z) where $x = \rho\cos(\phi), y = \rho\sin(\phi), \rho = \sqrt{x^2+y^2}, \phi = \operatorname{arc\,tg}(y/x)$; spherical (r,θ,ϕ) where $x = r\sin(\theta)\cos(\phi), y = r\sin(\theta)\sin(\phi), z = r\cos(\theta), r = \sqrt{x^2+y^2+z^2}, \theta = \operatorname{arc\,cos}(z/r), \phi = \operatorname{arc\,tg}(y/x)$.

Lame coefficients: $h_i^2 = \sum_j (\partial_{r_i'} r_j)^2$

Versors:$\partial_i\vec{r} = \vec{e_i}h_i$

Example: for cylindrical coordinates $h_\rho^2 = (\partial_\rho x)^2 + (\partial_\rho y)^2 + (\partial_\rho z)^2 = \cos^2(\phi) + \sin^2(\phi) + 0 = 1$

Integral implements: $\vec{dl} = h_idr_i\vec{e_i}, \vec{dA} = h_ih_jdr_idr_je_i \vec{\times} e_j, dV = h_ih_jh_kdr_idr_jdr_k$

Gradient: $\vec{\nabla}f = \frac{\vec{e_i}}{h_i}\partial_i f$

Divergence: $\vec{\nabla} \cdot \vec{A} = \frac{1}{h_1h_2h_3}\partial_i(h_jh_kA_i)$

Rotation: $\vec{\nabla} \times \vec{A} = \epsilon_{ijk}\frac{\vec{e_i}}{h_jh_k}\partial_j(h_kA_k)$

Laplasian: $\triangle f = \vec{\nabla} \cdot \vec{\nabla}f = \frac{1}{h_1h_2h_3}\partial_i\left(\frac{h_jh_k}{h_i}\partial_i f\right)$, for vector laplasian apply to each element

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=0, \vec{\nabla}\times\vec{\nabla}f=0, \vec{\nabla}\times(\vec{\nabla}\times\vec{A})=\vec{\nabla}(\vec{\nabla}\cdot\vec{A})-\triangle\vec{A}$

„Cuneiform“formula: $\nabla\cdot\nabla=\triangle$

Stokes theorem: $\oint_{\partial A}\vec{B}\cdot d\vec{l}=\iint_A(\vec{\nabla}\times\vec{B})\cdot d\vec{A}$

Gauss theorem: $\oiint_{\partial V}\vec{B}\cdot d\vec{A}=\iiint_V\vec{\nabla}\cdot\vec{B}dV$

Laplace polynomials: $P_l(x)=\dfrac{\partial_x^2(x^2-1)^l}{2^l l!}$

0.2 Electrodynamics

$$\text{Maxwell equations: }\begin{cases}\vec{\nabla}\cdot\vec{D}=\rho\\ \vec{\nabla}\cdot\vec{B}=0\\ \vec{\nabla}\times\vec{E}=-\partial_t\vec{B}\\ \vec{\nabla}\times\vec{H}=\vec{j}+\partial_t\vec{D}\end{cases}\qquad\text{(differential form)}$$

$$\begin{cases}\oiint\vec{D}d\vec{A}=\iiint\rho dV\\ \oiint\vec{B}d\vec{A}=0\\ \oint\vec{E}d\vec{l}=\iint-\partial_t\vec{B}d\vec{A}\\ \oint\vec{H}d\vec{l}=\iint(\vec{j}+\partial_t\vec{D})d\vec{A}\end{cases}\qquad\text{(integral form)}$$

$$\text{Distance notation: }\begin{cases}\vec{r}=\text{distance from zero coordinate to probe}\\ \vec{r'}=\text{distance from zero coordinate to object}\\ \vec{R}=\vec{r}-\vec{r'}=\text{distance from object to probe}\end{cases}$$

0.2.1 Electrostatics in vacuum

Coulomb law: $\vec{E}(\vec{r})=\dfrac{1}{4\pi\epsilon_0}\iiint\rho(\vec{r'})\dfrac{\vec{R}}{R^3}dV'$

Scalar potential: $U=\dfrac{1}{4\pi\epsilon_0}\iiint\dfrac{\rho(\vec{r'})}{R}dV'$

Electric field energy: $W_e=\frac{1}{2}\iiint\vec{D}\cdot\vec{E}dV'=\frac{1}{2}\iiint\rho U dV'$

Connection between fields and potentials, force and energy: $\vec{E}=-\nabla\vec{U}-\partial_t\vec{A}, \vec{B}=\vec{\nabla}\times\vec{A}, \vec{F}=-\nabla\vec{W}$

Poisson equation: $\triangle U=\rho$

Multipole approximation: $U=\dfrac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\dfrac{1}{r^{n+1}}\iiint\rho(\vec{r'})(r')^nP_n(\cos\angle(\vec{r},\vec{r'}))dV'$

Dipole moment (electric): $\vec{p}=\iiint\rho(\vec{r'})\vec{r'}dV'$

Electric dipole in field: $\vec{N} = \vec{p} \times \vec{E}, \vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$

Quadrupole moment:$Q_{ij} = \iiint \rho(r') \left(3r'_i r'_j - |r'|^2 \delta_{ij}\right) dV'$

Dipole field:$U_{dip} = \frac{1}{4\pi\epsilon_0 r^3}\vec{p} \cdot \vec{r}$

Quadrupole field:$U_{quad} = \frac{1}{4\pi\epsilon_0 r^5}\vec{r} \cdot \overline{Q} \cdot \vec{r}$

Condensator capacity: $C = Q/U$

Sphere image (method of images): $ab = R^2, \frac{q}{a} + \frac{q'}{R} = 0$

Solutions of Laplace equation - separation of variables

Laplace equation: $\triangle U = 0$

Cartesian coordinates (2D): $U = (A_0 x + B_0)(C_0 y + D_0)+$
$+ \sum_{n=1}^{\infty} \left(A_n \sin(k_{nx}x) + B_n \cos(k_{nx}x)\right) \left(C_n \sinh(k_{ny}y) + D_n \cosh(k_{ny}y)\right)$

Cylindrical coordinates (2D): $U = \log \frac{\rho}{A_0} + B_0+$
$+ \sum_{n=1}^{\infty} \left(A_n \rho^n + B_n \rho^{-n}\right) \left(C_n \sin n\phi + D_n \cos n\phi\right)$

Spherical coordinates (special case - rotational symmetry): $U = \sum_{n=0}^{\infty}(A_n r^n + B_n r^{-(n+1)})P_n(\cos\theta)$

0.2.2 Magnetostatics in vacuum

Biot-Savart law: $\vec{B} = \frac{\mu_0}{4\pi} \iiint \frac{\vec{I} \times \vec{R}}{R^2}$

Vector potential: $\vec{A} = \frac{\mu_0}{4\pi} \iiint \frac{\vec{I}(\vec{r'}}{R} dV'$

Magnetic dipole moment:$\vec{m} = \iint \vec{j} \times d\vec{A} = \frac{I}{2} \oint \vec{R} \times d\vec{R}, \vec{A_{dip}} = \frac{\mu_0}{4\pi r^2}\vec{m} \times \vec{r}$

Magnetic dipole in field:$\vec{N} = \vec{m} \times \vec{B}, \vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})$

Mutual inductivity:$L_{12} = \frac{\mu_0}{4\pi} \oint\!\!\oint \frac{d\vec{I}_1 \cdot d\vec{I}_2}{R}$

Self-inductivity:$L = \frac{\iint \vec{B} \cdot d\vec{A}}{I}$

Electromotoric force:$\varepsilon = -L\partial_t I$

Magnetic field energy:$W_m = \frac{1}{2} \iiint \vec{B} \cdot \vec{H} dV' = \frac{LI^2}{2}$

0.2.3 Electromagnetic materials

Material equations: $\vec{D} = \epsilon_0 \vec{E} + \vec{P}, \vec{B} = \mu_0(\vec{H} + \vec{M})$

Connection to optical properties:$\epsilon_0 \mu_0 = \frac{1}{c^2}, \epsilon\mu = n^2$

Ohm's law: $\vec{j} = \sigma \vec{E}, U = IR$

Bound charges: $\sigma_b = \vec{P} \cdot \vec{n}, \rho_b = -\vec{\nabla} \cdot \vec{P}$

Bound currents: $\vec{J_b} = \vec{\nabla} \times \vec{M}, \vec{K_b} = \vec{M} \times \vec{n}$

Boundary conditions - metals:$V_m = const, E_\parallel = 0, B_\perp = 0$

Boundary conditions - dielectrics:$E_{\parallel 1} = E_{\parallel 2}, D_{\perp 1} - D_{\perp 2} = \sigma_f$

Boundary conditions - magnetics:$B_{\perp 1} = B_{\perp 2}, H_{\parallel 1} - H_{\parallel 2} = \vec{K_f} \times \vec{n}$

0.2.4 Dynamics

Lorenz force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Electric current energy: $W_c = IU$

Continuity equation: $\vec{\nabla} \cdot \vec{J} + \partial_t \rho = 0$

Poynting vector:$\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_0}$

Maxwell density tensor:$T_{ij} = \epsilon_0 \left(E_i E_j - \delta_{ij} \dfrac{E^2}{2} \right) + \dfrac{1}{mu_0} \left(B_i B_j - \delta_{ij} \dfrac{B^2}{2} \right)$

Energy flow:$-\partial_t W = \partial_t \left[\iiint \epsilon_0 E^2 + \dfrac{B^2}{\mu_0} dV \right] + \oiint \vec{S} \cdot d\vec{A}$

Force in electromagnetic system:$\vec{F} = \oiint \overline{T} d\vec{A} - \dfrac{1}{c^2} \partial_t \iiint \vec{S} dV$

Connection between Maxwell tensor and Poynting vector:$\vec{\nabla} \cdot \overline{T} = c^2 \partial_t \vec{S}$

Impedance in transmission line:$Z_t = \sqrt{\dfrac{R + i\omega L}{G + i\omega C}}$

Retarded time:$t_r = t - \dfrac{|r - r'|}{c}$

Relativistic potentials:$U(\vec{r},t) = \frac{qc}{4\pi\epsilon_0}\frac{1}{|r - r'|c - r \vec{-} r' \cdot \vec{v}}, \vec{A}(\vec{r},t) = \frac{U\vec{v}}{c^2}$